EFFECT OF WORLD BANK'S TRAFFIC LIGHT SYSTEM ON INTERNATIONAL FINANCIAL FLOW TO DEVELOPING CONTRIES

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EFFECT OF WORLD BANK'S TRAFFIC LIGHT SYSTEM ON INTERNATIONAL FINANCIAL FLOW TO DEVELOPING CONTRIES

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In 2004, the World Bank (IDA) introduced the Debt Sustainability Framework (DSF) for low-income countries, so called "traffic light system", based on which IDA determines the grant/loan ratio to be allocated to each recipient country for each fiscal year. For instance, the country receives 100% grant aid, if the country is classified as "red light" (i.e. unsustainable debt level), under the traffic light system. The World Bank and IMF have been requesting the other institutions (aid donors, export credit agencies and private lenders) to comply with this system, in order to maintain developing countries' debt sustainability and to avoid another debt crisis. Since its introduction, there is a growing concern for some actual and potential shortcomings embedded in the system.

My dissertation attempts to address three major issues among them in each essay with strong emphasis on policy implications. The first issue is the free rider problem. Some of the other institutions are suspected to be not complying with the system, and to have been providing non-concessional loans to the "red light" country, by abusing their debt carrying capacity which is improved by grant aid from IDA and other donors. The first chapter develops a theoretical model of the free rider problem, and tests empirically for free riding. The second chapter examines the traffic light system's implicit assumption on the monotonic relationship between the debt stock and the default probability, and provides a theoretical model as well as empirical

evidence of non-monotonicity. Panel Logit/Probit analysis indicates that the relationship is “N-shaped” which implies that there exist a paradoxical zone in which default probability could decrease rather than increase with debt stock. The third chapter addresses the possibility that the traffic light system unduly constrains the ability of recipient countries to finance their development goals. The theoretical model and numerical simulations show what the sustainable debt level should be and how “red light” shock affects the long-run economic growth of the recipient country.

BIOGRAPHICAL SKETCH

The author was born on January 1st in 1975 in Okinawa, Japan. He was born as the second son between Choei and Hiroko Taira. The economic gap between Okinawa and mainland Japan has always been his theme to be considered.

At the age of eighteen, he moved to Tokyo to start his undergraduate study at Keio University, majoring in political science. In 1997, he came to Cornell to purse the master's degree. At first he started his study at Department of City and Regional Planning, then transferred to the field of Regional Science afterward.

After completing the master program, he gained the valuable experiences as an officer in a Japanese Official Development Assistance (ODA) organization (JBIC) from 2000 to 2008. In 2008, he once again came back to Cornell to pursue the doctoral degree in the field of Applied Economics and Management, majoring in development economics.

He is married to Kayoko and the father of two sons, Ryusei and Shozan.

To my wife, Kayoko

and

my sons, Ryusei and Shozan

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CHAPTER 1

SIGNALING EFFECT AND DYNAMIC BARGAINING UNDER THE WORLD BANK'S TRAFFIC LIGHT SYSTEM

1. Introduction

In 2004, the World Bank (IDA) introduced the debt sustainability framework (DSF) for low-income countries, so called "traffic light system", based on which IDA determines the grant/loan ratio to be allocated to each recipient country for each fiscal year. For instance, the country receives 100% grant aid, if the country is classified as "red light" (i.e. its debt level is unsustainable), under the traffic light system. The World Bank and IMF have been requesting the other institutions (aid donors, export credit agencies and private lenders) to comply with this system, in order to maintain developing countries' debt sustainability and to avoid another debt crises.

Since its introduction, there is a growing concern for the free rider problem. Namely, some of the other institutions are not complying with the system, and might have been providing non-concessional loans to the "red light" country, by abusing their debt carrying capacity which is improved by grant aid from IDA and other donors. However, based on some political considerations to so-called "emerging donors" such as China, Brazil, and so forth, the responsibility to avoid non-concessional loans was now shifted onto the borrower's side[1], which makes this problem's essentials hushed up. One of the pioneering criticism on this issue is raised by Tan (2006).

The main contribution of this paper is to provide both the theoretical framework and the empirical analysis on the free rider problem of the traffic light

[1] In 2006, IDA introduced the policy on non-concessional borrowing, based on which the non-complying borrower country is penalized by either the hardened terms of assistance or the reduction of the assistance volume from IDA.

system. To the author's knowledge, little studies, both in empirical and theoretical sense, have been conducted on the traffic light system so far. This paper examines how actually the traffic light system affects the behavior of the borrower countries/the donor agencies/the private lenders, and the international financial flow to developing countries.

This paper is organized as follows. The next section reviews the related literature and provides a motivation for the need for research on the signaling effect and the free rider issues of IDA's traffic light system. The third section describes the theoretical model. The fourth section presents the empirical results through the econometric analysis. The last section provides some policy implications and way forward.

2. Literature Review

Although there are many studies on the effects of the international financial institutions (IFIs)' involvement in the debt rescheduling negotiations among the private lenders and the borrower country, there have been a limited number of attempts to model the involvement of the bilateral official creditors.

Bulow and Rogoff (1988) analyzed multilateral negotiations for rescheduling sovereign debt by using dynamic bargaining theoretic framework. The analysis illustrates how various factors such as debtor's gain from trade and the level of world interest rate affect the relative bargaining power of three parties; debtor country, private lenders and creditor country. Bulow and Rogoff (1989) also presents a dynamic bargaining model of international lending, which extends to modeling the stochastic output and interest rate. Bulow et al. (1992) applied the Nash bargaining solution to the four-way rescheduling negotiations among the IFIs, the official bilateral creditors, the debtor country and the private banks, in the context of the aid to

the former Soviet blocs in Europe. Diwan and Rodrik (1992) analyzed the model in which the IFIs' comparative advantage is their capacity to enforce the borrower country to implement the conditionality for adjustment programs. This comparative advantage enhances the value of the three-way debt reduction agreement. Spiegel (1996) examines a concerted debt reduction deal between a sovereign debtor, a private creditor, and an official creditor, who is supposed to insure the deposits of the commercial bank. His model assumes the exogenous debtor nation, and shows that a weakening of the financial position of the commercial bank reduces the contribution of the commercial bank and increases that of the official creditor. Klimenko (2002) analyzed the effect of a debtor country's pattern of trade with commercial creditors' home countries on the outcome of debt-rescheduling negotiations, based on Bulow and Rogoff (1998) approach. The analysis reveals that a debtor country with more market power has greater leverage in a three-way debt-rescheduling negotiation that includes the debtor country, its creditors and the International Financial Institutions.

The most of the past works on modeling the behavior of the official creditors and the private lenders focus on emerging economies, and there are a few studies on specifically devoted to the low income countries' context. Mehta and Thapa (1991) develops the microeconomic model of the supply of bank credit to the least developed countries (LDCs) by integrating the notion of political bailout, in order to explain the lending behavior of the banks. Their model suggests that U.S. commercial banks have behaved responsibly in extending large loans at low interest rates to a small number of LDCs, if the banks perceived the U.S. government warranty for bailout in case of LDCs' default. Their model actually captures the free riding behavior of the U.S. commercial banks that anticipate U.S. government's bailout would constitute a warranty.

Although some of these previous studies provide the foundations for the

theoretical model to be examined in this paper, none of them provide the empirical analyses to examine how well their theoretical model represent the actual data. On the other hand, there are large volume of empirical studies, which sometimes lacks the rigorous theoretical model, on the international financial flow to the developing countries. Lee (1993) examines whether the credit ratings assigned by the lenders can be explained by a set of explanatory variables selected from the willingness of the LDCs to repay their debt service obligations. The results indicate that the credit ratings provide a reasonable measure of the borrowers' creditworthiness and also that there is a geographical contagion in assigning credit ratings. Demirguc-Kunt and Detragiache (1994) compares the shares of official creditors in external debts of four Latin American countries (Argentina, Brazil, Chile, and Venezuela) and four Asian countries (Indonesia, South Korea, the Philippines and Turkey) in 1973-1989, and found that the Asian countries paid considerably lower average interest rates on external debt because of official creditors low interest rates. Thus, the Asian countries obtained substantial subsides in the form of loans at below-market interest rates. Kim and Wu (2008) address how the sovereign credit ratings provided by independent ratings agencies affect international capital flow, based on dataset of sovereign credit ratings from Standard & Poor's from 1995-2003 for a cross-section of 51 emerging markets. One of the findings is that the long-term foreign currency ratings positively correlate with international financial inflow. Improvement of the ratings attracts international financial inflow. Papaioannou (2009) examined how institutional quality of the developing countries affects international lending to them, by using a large panel of country-pair financial flow. The results suggest that poorly performing institutions, such as weak protection of property rights, are major impediments to foreign bank capital flow. Hayakawa, Kimura, and Lee (2011) examines the political and financial risk components that matter most for the activities of multinational

corporations in 93 countries including 60 developing countries. Their finding is that even though the political risk significantly affects the foreign direct investment decisions, the financial risk of the host country, except for the exchange rate stability, is not considered seriously.

Thus, there are a few attempts to provide both the theoretical model the empirical analyses. Among them, Rose and Spiegel (2002) developed the theoretical model in which a sovereign debtor countries allocate its borrowing across different creditor nations, when default penalties are based on proportional losses in the bilateral gains from trade, then applied the gravity model to explain how the creditors systematically lend more to countries with which they share close trade links with empirical data. Bjornskov and Philipp (2010) explores a causal link between aid and debt repayment by modeling the loan agreement between the developing country government and the international financial sector as a negotiation of self-interested agents, where the presence of foreign aid affects the negotiation outcome. A set of panel estimates including 93 developing countries shows that foreign aid is strongly negatively associated with repayment incentives. Therefore, one of this paper's aims is to bridge the theory and the empirics.

3. Theoretical Model

The theoretical model is constructed, based on the approach of Bulow and Rogoff (1988), and Klimenko (2002), which is applied to the three-way bargaining framework among (i)borrower country, (ii)bilateral donor country, and (iii)private lenders. Since their models were applied to the context of debt rescheduling negotiation for the borrower country which is in sovereign default crisis, it has slightly different background in problem setting. However, the basic framework shares applicable structures. The basic setup is as follows;

Step 0 Multilateral Donor determines the traffic light ("red") of Borrower Country, and exit from game. (i.e. the multilateral aid donors are assumed to be exogenous.)

Step1 Borrower Country threatens Private Lenders and Bilateral Donor Country, by insisting that Borrower Country would default if new money from Private Lenders and Bilateral Donor Country is not provided.

Step 2 Private Lenders and Bilateral Donor Country threaten back Borrower Country, by insisting that they would resort to trade sanction on Borrower Country in case of default. (The trade sanction leads to the reduced export from Borrower Country.)

Step 3 Three parties bargain over their share of the welfare gains from avoiding the default.

Step 4 When the three parties agree on their share, Borrower Country exports its output and three parties receive their share. The same game continues in each period.

For Step 3, the Bilateral Donor Country is assumed to make the first proposal. When all three parties are impatient to reach an agreement (i.e. have some sort of discount factor), there exists a unique stationary subgame perfect Nash Equilibrium. See Figure 1-1.

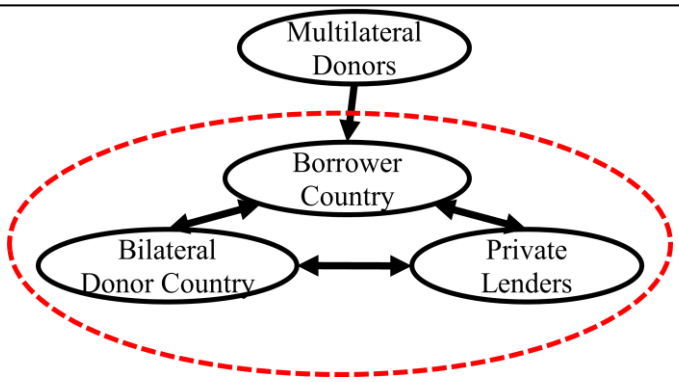

Figure 1-1: Three-way Bargaining Framework

3.1. Subgame Perfect Nash Equilibrium

Let ΔW be the total welfare gains from avoiding default, ΔW_B be the Borrower Country's welfare gains, and ΔW_D be the Bilateral Donor Country's welfare gains: $\Delta W = \Delta W_B + \Delta W_D$. Private Lenders are not supposed to receive benefits from welfare gains directly.

In order to find the equilibrium outcome, first step is to find each party's value of the stationary strategy from the second stage of negotiation, where each party has equal probability to make a proposal. Since all the subgames after the second stage have same structure, the following system of equations should be satisfied:

$$\left.\begin{aligned}
\widehat{V_B} &= \frac{1}{3}\left[\Delta W - \frac{1-\pi_D}{1+r_D}\widehat{V_D} - \frac{1-\pi_L}{1+r_L}\widehat{V_L}\right] + \frac{2}{3}\left[\frac{1-\pi_B}{1+r_B}\widehat{V_B}\right] \\
\widehat{V_L} &= \frac{1}{3}\left[\Delta W - \frac{1-\pi_B}{1+r_B}\widehat{V_B} - \frac{1-\pi_D}{1+r_D}\widehat{V_D}\right] + \frac{2}{3}\left[\frac{1-\pi_L}{1+r_L}\widehat{V_L}\right] \\
\widehat{V_D} &= \frac{1}{3}\left[\Delta W - \frac{1-\pi_B}{1+r_B}\widehat{V_B} - \frac{1-\pi_L}{1+r_L}\widehat{V_L}\right] + \frac{2}{3}\left[\frac{1-\pi_D}{1+r_D}\widehat{V_D}\right]
\end{aligned}\right\} \quad (1)$$

where $\widehat{V_B}$, $\widehat{V_L}$, $\widehat{V_D}$ are the stationary value after the second stage under the unanimous agreement scheme, and $r_B,\ r_L,\ r_D$ are the interest rate (assuming $r_D < r_L < r_B$), and $\pi_B,\ \pi_L,\ \pi_D$ are the subjective probability of default for the Borrower Country, and the Private Lenders, and the Donor Country respectively.

Thus, each party's discount factor has two components; the interest rate and the subjective probability of default. The latter is the key parameter, and is assumed to be a function of the traffic light. If the traffic light system delivers the "correct" signal, then the subjective probability of default would be higher as the light turns to be "red".

The system of equation (1) states that the proposer makes an offer so that the responders are indifferent between accepting the offer, the left-hand side, and rejecting it for a chance to make a counter-offer at the next period, the right-hand side.

By solving (1), which contains three unknowns and three equations, the following equilibrium values for the second stage are derived for each party.

$$\left.\begin{aligned}
\widehat{V_B} &= \frac{(1+r_B)(r_L+\pi_L)(r_D+\pi_D)}{(1+r_L)(r_B+\pi_B)(r_D+\pi_D)+(1+r_D)(r_B+\pi_B)(r_L+\pi_L)+(1+r_B)(r_L+\pi_L)(r_D+\pi_D)} \cdot \Delta W \\
\widehat{V_L} &= \frac{(1+r_L)(r_B+\pi_B)(r_D+\pi_D)}{(1+r_L)(r_B+\pi_B)(r_D+\pi_D)+(1+r_D)(r_B+\pi_B)(r_L+\pi_L)+(1+r_B)(r_L+\pi_L)(r_D+\pi_D)} \cdot \Delta W \\
\widehat{V_D} &= \frac{(1+r_D)(r_B+\pi_B)(r_L+\pi_L)}{(1+r_L)(r_B+\pi_B)(r_D+\pi_D)+(1+r_D)(r_B+\pi_B)(r_L+\pi_L)+(1+r_B)(r_L+\pi_L)(r_D+\pi_D)} \cdot \Delta W
\end{aligned}\right\} \quad (2)$$

Thus, at the first stage, the Donor Country's proposal is accepted by the Private Lenders and the Borrower Country if they are offered the shares of the discounted value of $\widehat{V_L}$ and $\widehat{V_B}$ respectively and the Donor Country receives the rest of the share, defined as below;

$$\left.\begin{aligned}
V_B &= \frac{1-\pi_B}{1+r_B} \cdot \widehat{V_B} \\
&= \frac{(1-\pi_B)(r_L+\pi_L)(r_D+\pi_D)}{(1+r_L)(r_B+\pi_B)(r_D+\pi_D)+(1+r_D)(r_B+\pi_B)(r_L+\pi_L)+(1+r_B)(r_L+\pi_L)(r_D+\pi_D)} \cdot \Delta W \\
V_L &= \frac{1-\pi_L}{1+r_L} \cdot \widehat{V_L} \\
&= \frac{(1-\pi_L)(r_B+\pi_B)(r_D+\pi_D)}{(1+r_L)(r_B+\pi_B)(r_D+\pi_D)+(1+r_D)(r_B+\pi_B)(r_L+\pi_L)+(1+r_B)(r_L+\pi_L)(r_D+\pi_D)} \cdot \Delta W \\
V_D &= \Delta W - V_B - V_L \\
&= \frac{(r_B+\pi_B)(r_L+\pi_L)(1+3r_D+2\pi_D)}{(1+r_L)(r_B+\pi_B)(r_D+\pi_D)+(1+r_D)(r_B+\pi_B)(r_L+\pi_L)+(1+r_B)(r_L+\pi_L)(r_D+\pi_D)} \cdot \Delta W
\end{aligned}\right\} \quad (3)$$

At equilibrium, the Borrower Country will export all its output each period, and the Private Lenders will receive its share, V_L, which corresponds to the maximum amount of the repayment that the Private Lenders can expect to receive from the Borrower Country. Therefore, the lending ceiling per period, L, is set at the discounted value of V_L as

$$\begin{aligned}
L &= \frac{1-\pi_L}{1+r_L} \cdot V_L \\
&= \frac{(1-\pi_L)^2(r_B+\pi_B)(r_D+\pi_D)}{(1+r_L)[(1+r_L)(r_B+\pi_B)(r_D+\pi_D)+(1+r_D)(r_B+\pi_B)(r_L+\pi_L)+(1+r_B)(r_L+\pi_L)(r_D+\pi_D)]} \cdot \Delta W
\end{aligned} \quad (4)$$

Thus, as its share, V_L, gets smaller, its lending ceiling becomes lower, since its lending ceiling is guaranteed by its share to be received.

The difference between the Donor Country's welfare, ΔW_D, and the Donor Country's share, V_D , defines the maximum amount of the grant aid, G, which is to be provided.

$$\begin{aligned} G &= \Delta W_D - V_D \\ &= \frac{(r_D+\pi_D)\{(r_B+\pi_B)(1-\pi_L)+(r_L+\pi_L)(1-\pi_B)\}}{(1+r_L)(r_B+\pi_B)(r_D+\pi_D)+(1+r_D)(r_B+\pi_B)(r_L+\pi_L)+(1+r_B)(r_L+\pi_L)(r_D+\pi_D)} \cdot \Delta W_D \\ &\quad - \frac{(r_B+\pi_B)(r_L+\pi_L)(1+3r_D+2\pi_D)}{(1+r_L)(r_B+\pi_B)(r_D+\pi_D)+(1+r_D)(r_B+\pi_B)(r_L+\pi_L)+(1+r_B)(r_L+\pi_L)(r_D+\pi_D)} \cdot \Delta W_B \end{aligned} \tag{5}$$

Thus, as its welfare gains, ΔW_D, is higher, the grant ceiling becomes higher, since the grant to be provide is supposed to be financed from the Donor Country's trade gains. On the other hand, as its share, V_D, gets smaller, the grant ceiling gets higher, since the grant ceiling is defined as the amount that the Donor Country has to compensate the Private Lender and the Borrower Country for avoiding default.

3.2. Comparative Statics

By comparative statics, it is useful to examine how the red light signal, R, affects the lending ceiling and the grant ceiling. For simplicity, let me assume that $\frac{\partial \pi_D}{\partial R} \geq 0$, $\frac{\partial \pi_B}{\partial R} = 0$, and $\frac{\partial \pi_L}{\partial R} \gtreqless 0$. First assumption is plausible, since the bilateral aid donors supported introduction of the traffic light system. Second assumption is reasonable, since the Borrower Country knows its own debt situation before the traffic light is revealed, which means that the traffic light is basically no surprising news to the Borrower Country.

3.2.1. Lending Ceiling

Let X be the denominator of (4). Then,

$$\frac{\partial L\ \ (\pi_D(R),\pi_L(R))}{\partial R} = \frac{\partial L}{\partial \pi_D}\cdot\frac{\partial \pi_D}{\partial R} + \frac{\partial L}{\partial \pi_L}\cdot\frac{\partial \pi_L}{\partial R}$$

$$\frac{\partial L}{\partial \pi_D} = \frac{(1-\pi_L)^2(r_B+\pi_B)X-(1-\pi_L)^2(r_B+\pi_B)(r_D+\pi_D)\cdot\{(1+r_L)^2(r_B+\pi_B)+(1+r_L)(1+r_B)(r_L+\pi_L)\}}{X^2}\cdot\Delta W$$

$$= \frac{\Delta W}{X^2}\cdot(1-\pi_L)^2(r_B+\pi_B)^2(r_L+\pi_L)(1+r_L)(1+r_D)$$

$$\frac{\partial L}{\partial \pi_L} =$$

$$\frac{(-2)(1-\pi_L)(r_B+\pi_B)(r_D+\pi_D)X-(1-\pi_L)^2(r_B+\pi_B)(r_D+\pi_D)\cdot(1+r_L)\{(1+r_D)(r_B+\pi_B)+(1+r_B)(r_D+\pi_D)\}}{X^2}$$

$$\cdot\Delta W$$

$$= -\frac{\Delta W}{X^2}\cdot(1-\pi_L)(r_B+\pi_B)(r_D+\pi_D)(1+r_L)[2\{(1+r_L)(r_B+\pi_B)(r_D+\pi_D)$$

$$+(1+r_D)(r_B+\pi_B)(r_L+\pi_L)+(1+r_D)(r_B+\pi_B)(r_L+\pi_L)\}$$

$$+(1-\pi_L)\{(1+r_D)(r_B+\pi_B)+(1+r_B)(r_D+\pi_D)\}]$$

$$\Longrightarrow \frac{\partial L\ \ (\pi_D(R),\pi_L(R))}{\partial R} = \frac{\Delta W}{X^2}\cdot(1-\pi_L)(r_B+\pi_B)(1+r_L)\cdot$$

$$\Big[(1-\pi_L)(r_B+\pi_B)(1+r_D)\Big\{\frac{\partial \pi_D}{\partial R}\cdot(r_L+\pi_L)-\frac{\partial \pi_L}{\partial R}\cdot(r_D+\pi_D)\Big\}$$

$$-\frac{\partial \pi_L}{\partial R}(r_D+\pi_D)[(1-\pi_L)(1+r_B)(r_D+\pi_D)+2\{(1+r_L)(r_B+\pi_B)(r_D+\pi_D)$$

$$+(1+r_D)(r_B+\pi_B)(r_L+\pi_L)+(1+r_B)(r_L+\pi_L)(r_D+\pi_D)\}]\Big] \qquad (6)$$

From the equation (6), $\frac{\partial L(\pi_D(R),\pi_L(R))}{\partial R} > 0$, if $\frac{\partial \pi_L}{\partial R} < 0$. On the other hand, if $\frac{\partial \pi_L}{\partial R} > 0$, then the sign of $\frac{\partial L(\pi_D(R),\pi_L(R))}{\partial R}$ is that of $\Big\{\frac{\partial \pi_D}{\partial R}\cdot(r_L+\pi_L)-\frac{\partial \pi_L}{\partial R}\cdot(r_D+\pi_D)\Big\}$. In order to see the sign of $\Big\{\frac{\partial \pi_D}{\partial R}\cdot(r_L+\pi_L)-\frac{\partial \pi_L}{\partial R}\cdot(r_D+\pi_D)\Big\}$, define α such that $\frac{\partial \pi_D}{\partial R} = \alpha\cdot\frac{\partial \pi_L}{\partial R}$. Here, α measures how the Donor Countries' subjective default probability respond to the traffic light relative to the Private Lenders' subjective default probability's change. Therefore, if the Donor Countries respond to the traffic light more aggressively than the Private Lenders, then $\alpha > 1$, and likewise, if the

Donor Countries respond to the traffic light less aggressively than the Private Lenders, then $0 < \alpha < 1$. Then,

$$\left\{\frac{\partial \pi_D}{\partial R} \cdot (r_L + \pi_L) - \frac{\partial \pi_L}{\partial R} \cdot (r_D + \pi_D)\right\} = \frac{\partial \pi_L}{\partial R} \cdot \{\alpha(r_L + \pi_L) - (r_D + \pi_D)\} \quad (7)$$

From the equation (7), if α takes larger value, then it is more likely that (7) would be positive. In other words, if the Donor Countries are expected to respond aggressively to the traffic light, then the Private Lenders are likely to increase their lending ceiling. Especially, if we assume $(r_D + \pi_D) > (r_L + \pi_L)$, then the equation (7) is always negative for $0 < \alpha < 1$, and could be positive if and only if $\alpha > 1$. Namely, if the Donor Countries are expected to respond only mildly to the traffic light, then the Private Lenders reduce their lending ceiling, but if the Donor Countries are expected to respond more aggressively to the traffic light than the Private Lenders, then the Private Lenders is more likely to increase their lending ceiling, which exactly is the free-riding by the Private Lenders.

To summarize the results, there are following three cases.

<case L1> If $\frac{\partial \pi_L}{\partial R} < 0$, then $\frac{\partial L}{\partial R} > 0$.

<case L2> If $\frac{\partial \pi_L}{\partial R} > 0$, and if α is small enough (such as $0 < \alpha < 1$), then $\frac{\partial L}{\partial R} < 0$.

<case L3> If $\frac{\partial \pi_L}{\partial R} > 0$, and if α is large enough (such as $\alpha > 1$), then $\frac{\partial L}{\partial R} > 0$.

<case L1> and <case L3> are the free-riding cases.

<case L1> is the case that the traffic light system is delivering the totally wrong signal to the Private Lenders. This case may be avoidable if the World Bank and IMF provide enough information of how the traffic light system works and could convince the Private Lenders accordingly.

<case L3> indicates the serious problem of the traffic light system. Even though the system delivers the correct signal to the Private Lenders, they may increase their lending if they expect that the Donor Countries would respond more aggressively.

3.2.2. Grant Ceiling

Let Z be the denominator of (5). Then,

$$\frac{\partial G(\pi_D(R),\pi_L(R))}{\partial R} = \frac{\partial G}{\partial \pi_D}\cdot\frac{\partial \pi_D}{\partial R} + \frac{\partial G}{\partial \pi_L}\cdot\frac{\partial \pi_L}{\partial R}$$

$$\frac{\partial G}{\partial \pi_D} = \frac{\{(1-\pi_L)(r_B+\pi_B)+(1-\pi_B)(r_L+\pi_L)\}Z-(r_D+\pi_D)\{(1-\pi_L)(r_B+\pi_B)+(1-\pi_B)(r_L+\pi_L)\}\{(1+r_L)(r_B+\pi_B)+(1+r_B)(r_L+\pi_L)\}}{Z^2}\cdot\Delta W_D$$

$$-\frac{\{2(r_B+\pi_B)(r_L+\pi_L)\}Z-(r_D+\pi_D)(r_L+\pi_L)(1+3r_D+2\pi_D)\{(1+r_L)(r_B+\pi_B)+(1+r_B)(r_L+\pi_L)\}}{Z^2}\Delta W_B$$

$$=\frac{\Delta W_D+\Delta W_B}{Z^2}\cdot(1+r_D)(r_B+\pi_B)(r_L+\pi_L)\{(1-\pi_L)(r_B+\pi_B)+(1-\pi_B)(r_L+\pi_L)\}$$

$$\frac{\partial G}{\partial \pi_L} = \frac{(r_D+\pi_D)\{-(r_B+\pi_B)+(1-\pi_B)\}Z-(r_D+\pi_D)\{(1-\pi_L)(r_B+\pi_B)+(1-\pi_B)(r_L+\pi_L)\}\{(1+r_D)(r_B+\pi_B)+(1+r_B)(r_D+\pi_D)\}}{Z^2}\cdot\Delta W_D$$

$$-\frac{\{(r_B+\pi_B)(1+3r_D+2\pi_D)\}Z-(r_B+\pi_B)(r_L+\pi_L)(1+3r_D+2\pi_D)\{(1+r_D)(r_B+\pi_B)+(1+r_B)(r_D+\pi_D)\}}{Z^2}\cdot\Delta W_B$$

$$=-\frac{\Delta W_D+\Delta W_B}{Z^2}\cdot(1+r_L)(r_D+D)(r_B+\pi_B)^2(1+3r_D+2\pi_D)$$

$$\Longrightarrow \frac{\partial G(\pi_D(R),\pi_L(R))}{\partial R}$$

$$=\frac{\Delta W_D+\Delta W_B}{Z^2}\cdot(r_B+\pi_B)\cdot\left[\frac{\partial \pi_D}{\partial R}\cdot(1+r_D)(r_L+\pi_L)\{(1-\pi_L)(r_B+\pi_B)+(1-\pi_B)(r_L+\pi_L)\}-\frac{\partial \pi_L}{\partial R}\cdot(1+r_L)(r_D+D)(r_B+\pi_B)(1+3r_D+2\pi_D)\right] \quad (8)$$

From the equation (8), $\frac{\partial G(\pi_D(R),\pi_L(R))}{\partial R} > 0$, if $\frac{\partial \pi_L}{\partial R} < 0$. On the other hand, if $\frac{\partial \pi_L}{\partial R} > 0$, then the sign of $\frac{\partial G(\pi_D(R),\pi_L(R))}{\partial R}$ is that of the bracket term. In order to see the

sign of the bracket term, define β such that $\beta \cdot \frac{\partial \pi_D}{\partial R} = \frac{\partial \pi_L}{\partial R}$. Here, β measures how the Private Lenders' subjective default probability respond to the traffic light relative to the Donor Countries' subjective default probability's change. Therefore, if the Private Lenders respond to the traffic light more aggressively than the Donor Countries, then $\beta > 1$, and likewise, if the Private Lenders respond to the traffic light less aggressively than the Donor Countries, then $0 < \beta < 1$.

From the equation (8), if β takes larger value, then it is more likely that (8) would be negative. In other words, if the Private Lenders are expected to respond aggressively to the traffic light, then the Donor Countries are likely to reduce their grant ceiling. Especially, if we assume $(r_D + \pi_D) > (r_L + \pi_L)$ and $(r_B + \pi_B) > (1 - \pi_B)$, then the equation (8) is always negative for $\beta > 1$, and could be positive if and only if $0 < \beta < 1$. Namely, if the Private Lenders are expected to respond aggressively to the traffic light, then the Donor Countries reduce their grant ceiling, but if the Private Lenders are expected to respond only mildly to the traffic light than the Donor Countries, then the Donor Countries is more likely to increase their grant ceiling.

To summarize the results, there are following three cases.

<case G1> If $\frac{\partial \pi_L}{\partial R} < 0$, then $\frac{\partial G}{\partial R} > 0$.

<case G2> If $\frac{\partial \pi_L}{\partial R} > 0$, and if β is large enough (such as $\beta > 1$),, then $\frac{\partial G}{\partial R} < 0$.

<case G3> If $\frac{\partial \pi_L}{\partial R} > 0$, and if β is small enough (such as $0 < \beta < 1$) then $\frac{\partial G}{\partial R} > 0$.

<case G1> and <case G3> are the free-riding cases, since it corresponds to <case L1> and <case L3> respectively, which is simply because $\beta = \frac{1}{\alpha}$ by definition.

Thus, increase in the lending ceiling by the Private Lenders occurs along with

increase in the grant ceiling by the Donor Countries, which implies that the Private Lenders extract resources from the Donor Countries.

4. Empirical Analysis

Before conducting the empirical analyses, let me summarize what is predicted by the theory, developed in the previous section. The results of the comparative statics are summarized in Table 1-1.

Table 1-1: Summary Table for Theoretical and Empirical Variables

Theoretical Variable	Expected Sign for L	Expected Sign for G	Empirical Variable
L			Private Lenders' Foreign Claims
G			ODA Grant Commitment
R	$+$ for large α	$+$ for small β	Red Light Dummy (1=Red Light)
	$-$ for small α	$-$ for large β	
α	$+$	$+$	Borrower's Aid Dependency (% of GNI)
β	$-$	$-$	Borrower's Sort-Term Debt (% of Total External Debt)
ΔW_D	$+$	$+$	Exports from Donor to Borrower (million US$)
ΔW_B	$+$	$-$	Imports to Donors from Borrowers (million US$)
r_D	$+$	$+$	Donor's Real Interest Rate (%)
r_B	$+$	$-$	Borrower's Real Interest Rate (%)
r_L	$-$	$-$	Lender's Real Interest Rate (%)
π_D	$+$*	$+$*	unobservable
π_B	0*	0*	unobservable
π_L	$\pm$*	$\pm$*	unobservable

* The expected sign is of the assumption.

The theory predicts how the lending ceiling as well as the grant ceiling is determined by the games among the three parties, the Private Lender, the Borrower Country, and the Donor Country, and the basic structure of the model describes how to split the welfare gains from avoiding default to three parties. Therefore, the higher the welfare gains are, the higher what they receive as their shares. For the following empirical analyses, let the trade flows between the Borrower Country and the Donor Country be the welfare gains. Thus, in case of the lending ceiling, if they are the important trade partners, then the lending ceiling becomes higher, which is represented by the positive expected sign of ΔW_D and ΔW_B on L in Table 1-1. On the other hand, the case of the grant ceiling is not straightforward. Since the grant ceiling is positively correlated with the trade benefits to the Donor Country from the equation (5), the expected sign of ΔW_D on G is positive in Table 1-1. However, because the Donor Country's share is negatively correlated with the grant ceiling, ΔW_B has the negative expected sign on G.

Due to the structure of the model, the more impatient the parties are in reaching a settlement (i.e. the higher r are), the smaller their shares become. This is represented by the positive expected sign of r_D and r_B, and the negative expected sign of r_L on the lending ceiling. In case of the grant ceiling, the other parties' impatience leads to the higher share of its own, which results in lower grant ceiling. Likewise higher impatience of its own reduces its share, which results in higher grant ceiling.

4.1. Data and Specification

The dataset consists of annual observations of 18 out of 24 OECD-DAC members and 59 out of 64 "IDA-only" countries for the period of 2000-2010. (Please see Appendix 1-3 for the sample countries list.) The data can be classified into (i)

provider-recipient country pair flow data, (ii) IDA's traffic light, and (iii) data of on other controls.

For (i) provider-recipient country pair flow data, the dataset includes there types of flow; export/import flow between the provider country and the recipient country, the official development assistance flow from the provider country to the recipient country, and the banking cross-border claims from the provider countries where the lenders reside to the recipient country. Consequently, the dataset contains 1,062 country pair (18 times 59). These are the dependent variables. For (ii) IDA's traffic light, dummy variable, which takes a value of 1 if the country is classified as "red light", was constructed. This is the key explanatory variable. Since the World Bank's fiscal year starts July and ends June, there is a six-month period mismatch to other data. Therefore, when estimating models, the current and the one period lagged values are to be included in the models. For (iii) data of on other controls, there are bilateral relationship data (such as distance, colonial relationship, and common language) as well as economic data of each country (such as real interest rate, GNI per capita, and GDP growth rate). Among them, the most important data are the aid dependency of the recipient country and the short-term debt ratio. The former is supposed to be a proxy of α defined in 3.2.1., and the latter is suppose to be a proxy of β defined in 3.2.2.. (Please see Appendix 1-2 for data sources.)

Since conditions that lead a pair of countries to be more interdependent in trade are likely to lead more private lending and grand aid flow between them, the model specification follows the gravity model of international trade. The specification is as follows;

$$y_{ij,t} = \beta_0 + \beta_1 Red_{j,t} + \beta_2 Red_{j,t-1} + \beta_3 Export_{ij,t} + \beta_4 Import_{ij,t} + \beta_5 Interest_{j,t} + \beta_6 GNI_{j,t} + \beta_7 GDP_{j,t} + \beta_8 AidDep_{j,t} + \beta_9 ShortD_{j,t} + \beta_{10} Others_{j,t}$$

$$+\beta_{11}Interest_{i,t} + \beta_{12}GNI_{i,t} + \beta_{13}GDP_{i,t} + \beta_{14}Others_{i,t}$$
$$+\beta_{15}ComLang_{ij} + \beta_{16}Colony_{ij} + \beta_{17}Distance_{ij} + \beta_{18}FixedEffect_{ij}$$
$$+ \varepsilon_{ij,t}$$

where the subscript *i* denotes the donor/lender country, *j* denotes the borrower country, and *t* denotes time. The variables are defined as;

$y_{ij,t}$: the private lending and the ODA grant commitment flow

Red: dummy that takes a value of 1 if the country is classified as "red light"

$Export$: export from donor/lender country to borrower country

$Import$: import by donor/lender country from borrower country

$Interest$: real interest rate

GNI: GNI per capita

GDP: GDP growth rate

$AidDep$: aid dependency defined as aid inflow % of GNI

$ShortD$: short-term debt (maturity with less than a year) % of total external debt stock

$Others$: other control variables

$ComLang$: dummy that takes a value of 1 if two countries share a common official language

$Colony$: dummy that takes a value of 1 if two countries have ever had a colonial link

$Distance$: distance between their Capital Cities

$FixedEffect$: time invariant fixed effect of each country pair

4.2. Analysis on Structural Break

Before conducting detailed analysis, it is necessary to verify that there exist the "structural break" when the IDA's traffic light system was introduced in 2004.

It is visually not obvious that there is the trend difference between the period before and after 2004. (See Figure 1-2.) In order to examine whether this is the case,

Chow test was conducted by separating the sample to Pre-IDA traffic light system period (2000-2003) and Post-IDA traffic light system period (2004-2010). Chow test is F-test to the null hypothesis that the coefficients on both periods are same. (i.e. there is no structural difference after the traffic light system was introduced.)

According to the results presented in Table 1-2, the null hypothesis is rejected in both cases of private lending and ODA grant as a dependent variable. Therefore, it is possible to conclude that there is the structural break when IDA's traffic light system was introduced.

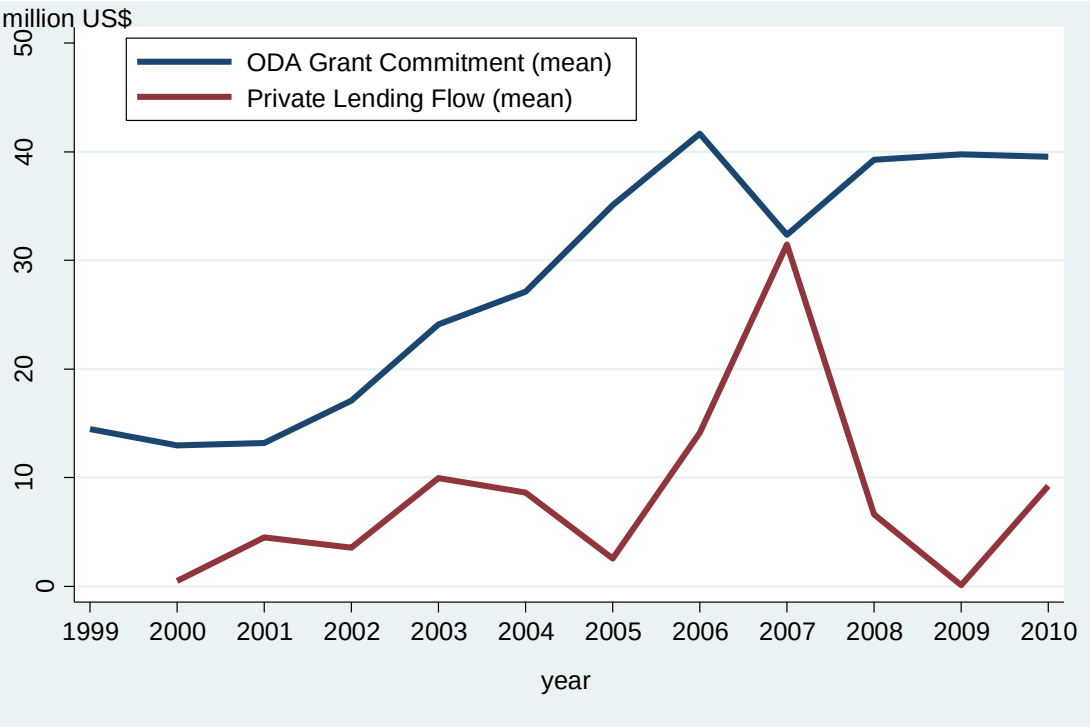

Figure 1-2: ODA Grant and Private Lending Flow

Table 1-2: Structural Break Analysis

X_{it} \ Y_{it}	Private Lenders' Foreign Claims		ODA Grant Commitment	
	Pre-IDA *2000-2003*	Post-IDA *2004-2008*	Pre-IDA *2000-2003*	Post-IDA *2004-2008*
constant	-22.904 [0.460]	-13.730 [0.217]	-25.787 [0.003]***	-19.699 [0.059]*
Exports from Donor to Borrower (million US$)	0.008 [0.489]	0.069 [0.000]***	0.055 [0.000]***	0.036 [0.000]***
Imports to Donors from Borrowers (million US$)	0.004 [0.388]	-0.001 [0.419]	-0.001 [0.550]	-0.002 [0.215]
Borrower's Real Interest Rate (%)	0.472 [0.010]***	-0.163 [0.499]	-0.010 [0.856]	-0.526 [0.012]**
Borrower's GNI per capita (current US$)	0.004 [0.403]	0.003 [0.213]	-0.002 [0.357]	-0.010 [0.000]***
Borrower's GDP Growth Rate (annual %)	-0.727 [0.028]**	1.346 [0.006]***	-0.057 [0.668]	-0.402 [0.361]
Borrower's Aid Dependency (% of GNI)	0.295 [0.255]	0.937 [0.000]***	0.653 [0.000]***	0.242 [0.036]**
Borrower's Sort-Term Debt (% of Total External Debt)	0.240 [0.230]	0.354 [0.054]**	-0.197 [0.021]**	-0.029 [0.864]
Borrower's Total Reserves (% of Total External Debt)	-0.019 [0.900]	-0.094 [0.007]***	-0.042 [0.471]	0.125 [0.000]***
Borrower's Population	2.28e-07 [0.002]***	1.24e-07 [0.897]	1.74e-07 [0.000]***	4.42e-07 [0.000]***
Borrower's Terms of Trade	0.140 [0.653]	-0.008 [0.897]	0.170 [0.040]**	0.084 [0.112]
Donor's Real Interest Rate (%)	-0.071 [0.930]	-2.232 [0.019]**	-0.436 [0.143]	-0.886 [0.326]
Donor's GNI per capita (current US$)	-0.0001 [0.638]	-0.00002 [0.917]	0.0004 [0.005]***	0.0004 [0.031]**
Donor's GDP growth rate (annual %)	1.962 [0.148]	0.524 [0.515]	0.192 [0.686]	0.416 [0.543]
Donor's Population	-2.45e-08 [0.459]	-1.33e-08 [0.647]	1.06e-07 [0.000]***	3.52e-07 [0.000]***
R^2	0.02	0.10	0.18	0.14
observation	1,369	3,068	2,443	3,954
Chow test (H_0: no structural break at 2004)	2.64 [0.001]***		9.21 [0.000]**	

Note: p-value is reported in [ ] and *, **, *** denotes significance at the 10%, 5%, and 1% level respectively

4.3. Country Pair Panel Analysis

Table 1-3 presents the estimated results of country pair panel data analysis.

Table 1-3: Country Pair Panel Estimation

X_{it} \ Y_{it}	Private Lenders' Foreign Claims			ODA Grant Commitment			
	Model (1) Fixed-Effect	Model (2) Random-Effect	Model (3) IV	Model (4) Fixed-Effect	Model (5) Random-Effect	Model (6) Random-Effect	Model (7) IV
Red Light dummy at current period (1=Red Light Country)	17.376 [0.098]*	10.970 [0.184]	17.484 [0.097]*	1.800 [0.822]	-11.243 [0.105]	-11.897 [0.084]*	1.621 [0.840]
Red Light dummy at previous period (1=Red Light Country)	-10.089 [0.296]	-18.108 [0.026]**	-10.654 [0.272]	0.515 [0.944]	-2.834 [0.674]	-4.379 [0.515]	0.934 [0.900]
Exports from Donor to Borrower (million US$)	-0.015 [0.433]	0.070 [0.000]***	0.049 [0.410]	-0.050 [0.001]***	0.023 [0.001]***	0.019 [0.006]***	-0.103 [0.032]**
Imports to Donors from Borrowers (million US$)	0.007 [0.303]	-0.003 [0.096]*	-	-0.006 [0.288]	0.001 [0.625]	0.001 [0.455]	-
Borrower's Real Interest Rate (%)	0.081 [0.835]	-0.165 [0.525]	0.139 [0.725]	-1.650 [0.000]***	-0.638 [0.006]***	-0.607 [0.008]***	-1.676 [0.000]***
Borrower's GNI per capita (current US$)	-0.018 [0.081]*	0.001 [0.623]	-0.023 [0.044]**	-0.018 [0.025]**	-0.013 [0.000]***	-0.012 [0.000]***	-0.015 [0.091]*
Borrower's GDP Growth Rate (annual %)	1.802 [0.011]**	2.000 [0.001]***	1.894 [0.009]***	-0.784 [0.169]	-0.369 [0.465]	-0.233 [0.643]	-0.842 [0.143]
Borrower's Aid Dependency (% of GNI)	1.266 [0.000]***	1.019 [0.000]**	1.239 [0.000]***	0.650 [0.001]***	0.354 [0.009]***	0.281 [0.035]**	0.669 [0.001]***
Borrower's Short-Term Debt (% of Total External Debt)	-1.188 [0.007]***	0.195 [0.361]	-1.195 [0.007]***	-0.481 [0.180]	-0.128 [0.539]	-0.050 [0.805]	-0.445 [0.219]
Borrower's Total Reserves (% of Total External Debt)	0.379 [0.005]***	-0.056 [0.136]	0.349 [0.009]***	0.196 [0.046]**	0.154 [0.000]***	0.129 [0.000]***	0.204 [0.038]**
Borrower's Population	-8.17e-06 [0.000]***	-9.90e-09 [0.911]	-9.43e-06 [0.000]***	-0.00002 [0.000]***	3.39e-07 [0.000]***	3.82e-07 [0.000]***	-0.00001 [0.000]***
Borrower's Terms of Trade	0.284 [0.177]	-0.014 [0.831]	0.286 [0.174]	-0.230 [0.126]	0.050 [0.424]	0.053 [0.386]	-0.227 [0.133]
Donor's Real Interest Rate (%)	0.916 [0.653]	-1.741 [0.102]	0.673 [0.744]	-2.088 [0.206]	-1.168 [0.276]	-1.197 [0.264]	-1.847 [0.270]
Donor's GNI per capita (current US$)	0.002 [0.015]**	-0.0008 [0.687]	0.003 [0.008]***	0.003 [0.000]***	0.0004 [0.141]	0.0004 [0.058]*	0.003 [0.000]***
Donor's GDP growth rate (annual %)	0.450 [0.655]	0.495 [0.567]	0.446 [0.659]	-0.767 [0.289]	0.448 [0.508]	0.445 [0.511]	-0.763 [0.292]
Donor's Population	-2.07e-06 [0.156]	-1.07e-08 [0.735]	-2.00e-06 [0.322]	7.03e-06 [0.000]***	3.88e-07 [0.000]***	3.85e-07 [0.000]***	6.99e-06 [0.000]***
Common Official Language dummy (1= if common official language)						27.502 [0.000]***	
Colonial Link dummy (1=if colonial link)						46.455 [0.000]***	
Distance						-0.001 [0.276]	
constant	154.047 [0.156]	-7.824 [0.531]	160.525 [0.130]	-61.417 [0.456]	-6.419 [0.629]	-11.121 [0.421]	-75.925 [0.367]
R^2	0.002	0.087	0.000	0.001	0.150	0.173	0.001
F-test / Wald-test (H_0: all $\beta = 0$)	5.50 [0.000]***	241.20 [0.000]***	125.89 [0.000]***	13.55 [0.000]***	390.99 [0.000]***	500.40 [0.000]***	630.98 [0.000]***
Hausman test (H_0: Random-Effect Model is correct.)	64.43 [0.000]***		-	57.06 [0.000]***		-	-
observation	2,558	2,558	2,558	3,306	3,306	3,306	3,306

Note: p-value is reported in [ ] and *, **, *** denotes significance at the 10%, 5%, and 1% level respectively

Model (1) reports estimates of the fixed-effects model for the private lending flow. Dummy variable of "red light" at current period enters with a positive coefficient which is statistically significant. It indicates that the private lenders increase their lending to the borrower if its traffic light is red. This is clearly the evidence of free-riding by the private lenders. In addition, the borrower's aid dependency, which is supposed to be a proxy of α defined in 3.2.1., has a positive and statistically significant coefficient and the borrower's short-term debt, which is supposed to be a proxy of β defined in 3.2.2., has a negative and statistically significant coefficient, as predicted by the result of comparative statics. This also supports the evidence that the private lenders are free-riding on the international aid inflow. On the other hand, dummy variable of "red light" at previous period negative coefficient which is not statistically significant. Exports from the donor to the borrower has a negative and statistically insignificant coefficient, which is opposite to the prediction by the theoretical model developed in the previous section. The coefficient on borrower's as well as donor's real interest rate are not statistically significant.

Model (2) reports estimates of the random-effects model for the private lending flow. Since the random-effects model was rejected by the Hausman test, it is not meaningful to go over the result of Model (2).

Model (4) reports estimates of the fixed-effects model for the ODA grant commitment flow. Neither of dummy variable of "red light" at current period or at previous period has a statistically significant result, which implies that the bilateral donor countries do not respond to the traffic light. The borrower's aid dependency, which is supposed to be a proxy of α defined in 3.2.1., has a positive and statistically significant coefficient, but the borrower's short-term debt, which is supposed to be a proxy of β defined in 3.2.2., has a negative but statistically insignificant coefficient.

Exports from the donor to the borrower has a negative and statistically significant coefficient, which is against to what is predicted by the theoretical model. The coefficient on the borrower's real interest rate is negative and statistically significant, as predicted by the theoretical model.

Model (5) reports estimates of the random-effects model for the ODA grant commitment flow. Like Model (2), the random-effects model was rejected by the Hausman test, it is not meaningful to go over the result of Model (5).

The results of Model (4) seem mixed and difficult to interpret. One of the reasons behind this could be that the bilateral donor countries put more weight on other factors during their grant allocation decision making. As an example, Model (6) presents estimates which includes the bilateral relationship variables; common language, colonial link, and distance. Since the random-effects model is rejected, the result is only for reference. The language and colonial links enters positive and statistically significant coefficients. Therefore, the bilateral donor countries take into account its cultural and historical relationship. And these effects are absorbed into the country pair fixed effects in Model (4).

The causality between the export/import flow and the private lending flow and/or the ODA grant flow seems opposite, especially for the private lending flow, from what the theory predicts. For instance, the private lending might be associated with the trade credit which supports export/import. Therefore, the private lending would promote larger volume of trade flow. Or if the provided ODA grant contributes to the recipient country's growth, the ODA grant might increase the recipient export flow. If this is the case, the causality is indeed reversed. In order to manage this problem of endogeneity, the two-staged least squares estimation by using instrument variables (IV model) is considered in Model (3) and Model (7). As the instruments, (i) the import by donor from the borrower, (ii) dummy variable of common language,

(iii) dummy variable of colonial links, and (iv) distance between the borrower country and the donor country are used. The result for Model (3) reversed the sign of coefficient on the export flow into positive, as predicted by the theoretical model, although its statistical significance remains low. The basic results remain same as Model (4), while the exports lose its explanatory powers.

Table 1-4 explores the sensitivity of the results, for the ODA-related variables, by using different dependent variables. For Model (8) through (11), the traffic light does not affect the dependent variables.

Model (8) uses the Grant Commitment as the dependent variable but excluding France, Germany and Japan, which are the largest ODA loan provider among OECD-DAC. Model (9) uses the Grant Commitment of these three countries only.

Model (10) uses the ODA Loan Commitment as the dependent variable for all samples of Post-IDA period. Model (11) uses the ODA Loan Commitment of France, Germany and Japan.

Model (12) uses the ODA Grant Ratio, defined as the ODA Grant Commitment divided by the ODA Total Commitment, for all samples of Post-IDA period. Model (13) uses the ODA Grant Ratio of France, Germany and Japan. For both models, the Grant Ratio decreases in response to the red light signal, which is hard to interpret. One of the possible interpretations is that the ODA Loan might increase relative to the ODA Grant, which results in the decreasing Grant Ratio. However, this interpretation is not supported by the statistical analysis, since reactions of both the ODA Loan and the ODA Grant are not statistically significant.

Table 1-4: Sensitivity Check for ODA Variables

X_{it} / Y_{it}	ODA Grant Commitment		ODA Loan Commitment		ODA Grant Ratio	
	Model (8) Excluding France, Germany and Japan	Model (9) France, Germany and Japan only	Model (10) All Samples	Model (11) France, Germany and Japan only	Model (12) All Samples	Model (13) France, Germany and Japan only
Red Light dummy at current period (1=Red Light Country)	-0.324 [0.749]	2.683 [0.914]	0.636 [0.713]	2.839 [0.694]	-0.017 [0.094]*	-0.072 [0.029]**
Red Light dummy at previous period (1=Red Light Country)	2.746 [0.682]	-10.818 [0.638]	1.045 [0.513]	-1.680 [0.802]	0.007 [0.455]	0.030 [0.333]
Exports from Donor to Borrower (million US$)	-0.054 [0.000]***	-0.029 [0.613]	0.019 [0.000]***	0.072 [0.000]***	-0.00002 [0.311]	-0.00002 [0.792]
Imports to Donors from Borrowers (million US$)	-0.005 [0.273]	-0.001 [0.974]	-0.0003 [0.838]	-0.025 [0.068]*	3.27e-06 [0.649]	0.00003 [0.609]
Borrower's Real Interest Rate (%)	-1.109 [0.000]***	-3.566 [0.000]***	0.007 [0.912]	0.073 [0.790]	0.0003 [0.446]	0.00003 [0.411]
Borrower's GNI per capita (current US$)	-0.013 [0.077]*	-0.023 [0.319]	0.002 [0.229]	0.003 [0.616]	-0.000002 [0.135]	-0.00002 [0.553]
Borrower's GDP Growth Rate (annual %)	-0.540 [0.300]	-1.642 [0.345]	0.081 [0.510]	0.365 [0.472]	-0.001 [0.070]*	-0.007 [0.005]***
Borrower's Aid Dependency (% of GNI)	0.292 [0.103]	2.112 [0.001]***	0.021 [0.620]	0.031 [0.869]	-0.0004 [0.099]*	-0.002 [0.021]**
Borrower's Short-Term Debt (% of Total External Debt)	-0.180 [0.578]	-1.696 [0.136]	0.004 [0.959]	0.209 [0.528]	0.0006 [0.162]	0.002 [0.300]
Borrower's Total Reserves (% of Total External Debt)	0.122 [0.166]	0.653 [0.050]**	0.042 [0.049]**	0.081 [0.403]	-0.00001 [0.930]	-0.0001 [0.847]
Borrower's Population	-7.46e-06 [0.000]***	-0.00005 [0.000]***	7.87e-07 [0.015]**	4.57e-06 [0.002]***	-2.39e-09 [0.202]	-1.44e-08 [0.029]**
Borrower's Terms of Trade	-0.067 [0.621]	-0.853 [0.084]*	0.020 [0.546]	-0.035 [0.810]	-0.0002 [0.201]	-0.001 [0.206]
Donor's Real Interest Rate (%)	-0.795 [0.554]	0.836 [0.971]	0.562 [0.115]	21.852 [0.001]*	0.001 [0.697]	--0.038 [0.217]
Donor's GNI per capita (current US$)	0.002 [0.015]**	0.008 [0.007]***	-0.0002 [0.310]	-0.002 [0.055]*	6.52e-07 [0.485]	8.28e-06 [0.044]**
Donor's GDP growth rate (annual %)	-0.906 [0.177]	-0.533 [0.832]	0.383 [0.014]**	0.285 [0.697]	-0.001 [0.078]*	0.0002 [0.951]
Donor's Population	6.04e-06 [0.000]***	-0.00001 [0.419]	-3.05e-07 [0.316]	1.55e-06 [0.721]	9.22e-10 [0.601]	-1.67e-08 [0.399]
constant	-106.51 [0.077]*	1,946.69 [0.146]	-2.621 [0.883]	-193.05 [0.621]	0.999 [0.000]***	2.626 [0.140]
R^2	0.032	0.085	0.015	0.144	0.001	0.012
F-test / Wald-test (H_0: all $\beta = 0$)	6.82 [0.000]***	9.70 [0.000]***	4.19 [0.000]***	4.20 [0.000]***	1.48 [0.099]*	1.99 [0.012]**
observation	2,627	679	3,315	679	3,306	679

Note: p-value is reported in [ ] and *, **, *** denotes significance at the 10%, 5%, and 1% level respectively

4.4. Robustness Checks

Table 1-5 presents the estimated results of robustness checks for the private lending flows.

Model (14) use the log transformation of the level data, with fixed-effect. Specifically, the private lenders' foreign claims, the export, the import, GNI per capita, and population are transformed into log. The red light dummy at current period loses statistical significance. The aid dependency variable remains its statistical significance.

Model (15) explores the case of autocorrelation of the error term. The red light dummy at current period maintains the positive sign but lose its statistical significance. The aid dependency variable also loses its statistical significance.

Model (16) includes the one-period lagged ODA Grant Commitment to the explanatory variables. The red light dummy at current period as well as the lagged ODA Grant Commitment are found to be positive and statistically significant, while the aid dependency variable loses its statistical significance. The correlation between the lagged ODA Grant Commitment and the aid dependency variables seems to make the result noisy to some extent.

Model (17) explores the dynamic model by adding lagged dependent variable as regressors. The consistent estimator is derived from Arellano and Bond's GMM method. The red light dummy at current period remains to be positive and its statistical significance. The aid dependency variable loses its statistical significance. The lagged private lending flow is positive and statistically significant, which indicates that the private lenders keep its lending flow pattern.

Model (18) and Model (19) presents the results obtained by the Generalized Least Squares approach. Model (18) assumes heteroscedastic but uncorrelated error term, and Model (19) assumes heteroscedastic and autocorrelated error term for each country-pair. For both models, the red light dummy at current period as well as the aid

dependency are found to be positive and statistically significant, as predicted by the theory. In addition to that, however, the red light dummy at previous period is found to be negative, and the short-term debt is found to be positive, both of which are statistically significant. These results are against the theory. Especially, the positive sign of the short-term debt is hard to interpret. On the other hand, the negative sign of the red light dummy at previous period could be interpreted as follows. The private lenders front-load their lending flow to the borrower once its traffic light is revealed to be red for that period, which is represented by the positive sign of the red light dummy at current period, then wait and see how the traffic light of the next period would be, which is represented by the negative sign of the red light dummy at previous period. In summary, for the private lending flow, the coefficient on the red light dummy at current period, and the borrower's aid dependency maintain the expected positive sign and statistical significance in most of the alternative specifications.

Table 1-6 presents the estimated results of robustness checks for the ODA Grant Commitment flow. Each model is same as the one applied in Table 1-5. The results are mixed. In some specifications, the coefficient on the red light dummy at current period is rather negative and statistically significant, although the aid dependency and the short-term debt maintain their expected sign in most of cases. In summary, the results indicate the weak stability of the model for the ODA Grant Commitment.

Table 1-5: Robustness Checks for Private Lenders' Foreign Claims

X_{it} / Y_{it}	Private Lenders' Foreign Claims					
	Model (14) Fixed-Effect, log transformation	Model (15) Fixed-Effect, AR(1)	Model (16) Fixed-Effect, Including lagged ODA Grant	Model (17) Dynamic	Model (18) Heteroscedastic, Uncorrelated Error	Model (19) Heteroscedastic, Panel Specific AR(1) Error
Red Light dummy at current period (1=Red Light Country)	0.212 [0.454]	10.534 [0.458]	17.050 [0.011]**	16.258 [0.051]*	1.458 [0.015]**	5.617 [0.000]***
Red Light dummy at previous period (1=Red Light Country)	-0.292 [0.121]	-11.536 [0.363]	-8.944 [0.196]	-14.451 [0.232]	-2.283 [0.001]***	-8.954 [0.000]***
Exports from Donor to Borrower (million US$)	0.160 # [0.266]	-0.045 [0.071]*	-0.015 [0.852]	-0.077 [0.326]	0.016 [0.002]***	0.065 [0.000]***
Imports to Donors from Borrowers (million US$)	0.147 # [0.002]***	0.003 [0.707]	0.008 [0.163]	0.005 [0.770]	-0.0004 [0.676]	0.002 [0.042]**
Borrower's Real Interest Rate (%)	0.008 [0.326]	0.093 [0.840]	0.127 [0.778]	0.148 [0.750]	-0.010 [0.688]	-0.168 [0.000]***
Borrower's GNI per capita (current US$)	-0.006 # [0.993]	-0.008 [0.627]	-0.023 [0.098]*	-0.008 [0.591]	0.0003 [0.318]	0.001 [0.002]***
Borrower's GDP Growth Rate (annual %)	0.077 [0.000]***	1.711 [0.066]*	2.231 [0.100]*	1.571 [0.148]	0.189 [0.006]***	1.250 [0.000]***
Borrower's Aid Dependency (% of GNI)	0.004 [0.044]**	0.302 [0.292]	0.909 [0.230]	0.758 [0.262]	0.115 [0.001]***	0.931 [0.000]***
Borrower's Short-Term Debt (% of Total External Debt)	0.013 [0.083]*	-0.339 [0.544]	-1.773 [0.246]	-0.426 [0.710]	0.021 [0.447]	0.072 [0.000]***
Borrower's Total Reserves (% of Total External Debt)	0.009 [0.001]***	0.168 [0.339]	0.364 [0.093]*	0.122 [0.509]	-0.006 [0.268]	-0.053 [0.000]***
Borrower's Population	1.023 # [0.723]	-9.56e-06 [0.005]***	-8.29e-06 [0.014]**	-5.58e-06 [0.036]**	-1.41e-09 [0.860]	-1.06e-08 [0.189]
Borrower's Terms of Trade	-0.002 [0.652]	0.465 [0.077]*	0.303 [0.218]	0.391 [0.198]	-0.008 [0.203]	-0.017 [0.002]***
Donor's Real Interest Rate (%)	0.061 [0.308]	0.362 [0.877]	1.770 [0.469]	2.208 [0.288]	-0.223 [0.007]***	-0.747 [0.000]***
Donor's GNI per capita (current US$)	0.306 # [0.813]	0.001 [0.343]	0.003 [0.005]**	0.001 [0.239]	5.17e-06 [0.764]	0.00003 [0.175]
Donor's GDP growth rate (annual %)	-0.050 [0.067]*	0.533 [0.622]	0.718 [0.437]	2.002 [0.070]*	0.092 [0.227]	0.332 [0.000]***
Donor's Population	2.141 # [0.795]	-2.54e-06 [0.353]	-2.17e-06 [0.283]	-3.48e-06 [0.243]	-1.15e-09 [0.749]	4.95e-09 [0.173]
lagged Private Lenders' Foreign Claims				0.154 [0.040]**		
lagged ODA Grant Commitment			0.069 [0.059]*			
constant	-56.50 [0.656]	249.18 [0.045]**	167.37 [0.258]	207.74 [0.203]	-1.027 [0.371]	-10.524 [0.000]***
R^2	0.127	0.001	0.001	-	-	-
F-test / Wald-test (H_0: all $\beta = 0$)	5.22 [0.000]***	2.13 [0.006]***	1.99 [0.011]**	32.91 [0.012]**	35.52 [0.003]***	12,603.24 [0.000]***
observation	701	1,972	2,111	1,868	2,558	2,496

Note: p-value is reported in [ ] and *, **, *** denotes significance at the 10%, 5%, and 1% level respectively
log transformed.

Table 1-6: Robustness Checks for ODA Grant Commitment

X_{it} / Y_{it}	ODA Grant Commitment					
	Model (20) Fixed-Effect, log transformation	Model (21) Fixed-Effect, AR(1)	Model (22) Fixed-Effect, Including lagged Private Lending	Model (23) Dynamic	Model (24) Heteroscedastic, Uncorrelated Error	Model (25) Heteroscedastic, Panel Specific AR(1) Error
Red Light dummy at current period (1=Red Light Country)	-0.032 [0.771]	-2.138 [0.729]	-0.781 [0.898]	-6.349 [0.122]	-3.491 [0.000]***	-7.577 [0.000]***
Red Light dummy at previous period (1=Red Light Country)	0.137 [0.166]	0.090 [0.987]	-0.391 [0.956]	-4.063 [0.552]	-3.188 [0.000]***	-1.235 [0.035]**
Exports from Donor to Borrower (million US$)	-0.019 # [0.669]	-0.005 [0.702]	-0.050 [0.152]	-0.045 [0.210]	0.031 [0.000]***	0.019 [0.000]***
Imports to Donors from Borrowers (million US$)	-0.004 # [0.863]	0.0002 [0.946]	-0.005 [0.384]	-0.001 [0.783]	-0.0001 [0.954]	0.001 [0.353]
Borrower's Real Interest Rate (%)	-0.013 [0.002]***	-0.091 [0.654]	-2.116 [0.004]***	-1.535 [0.016]**	-0.214 [0.000]***	-0.308 [0.000]***
Borrower's GNI per capita (current US$)	-0.001 # [0.000]***	0.0002 [0.981]	-0.021 [0.010]***	-0.016 [0.050]**	-0.006 [0.000]***	-0.008 [0.000]***
Borrower's GDP Growth Rate (annual %)	0.006 [0.454]	0.469 [0.261]	-0.705 [0.107]	-0.389 [0.140]	-0.025 [0.617]	-0.179 [0.000]***
Borrower's Aid Dependency (% of GNI)	0.013 [0.000]***	0.427 [0.002]***	0.867 [0.008]***	1.092 [0.007]***	0.116 [0.000]***	0.203 [0.000]***
Borrower's Short-Term Debt (% of Total External Debt)	-0.011 [0.101]	-0.478 [0.076]*	-0.607 [0.117]	-1.221 [0.062]	0.017 [0.430]***	0.012 [0.656]
Borrower's Total Reserves (% of Total External Debt)	0.004 [0.003]**	-0.207 [0.003]***	0.071 [0.642]	-0.168 [0.262]	0.027 [0.000]***	0.091 [0.000]***
Borrower's Population	-2.324 # [0.041**]	6.02e-06 [0.000]***	-0.00002 [0.018]**	-0.00001 [0.019]**	2.23e-07 [0.000]***	4.52e-07 [0.000]***
Borrower's Terms of Trade	0.002 [0.442]	-0.020 [0.856]	-0.372 [0.040]	-0.273 [0.068]*	0.026 [0.000]***	0.033 [0.000]***
Donor's Real Interest Rate (%)	0.056 [0.003]***	-1.216 [0.233]	-3.290 [0.103]	-3.339 [0.087]*	-0.571 [0.000]***	-0.216 [0.024]**
Donor's GNI per capita (current US$)	0.595 # [0.245]	-0.001 [0.337]	0.003 [0.028]**	0.003 [0.005]***	-0.0002 [0.000]***	0.0003 [0.000]***
Donor's GDP growth rate (annual %)	-0.019 [0.036]**	-0.841 [0.055]*	0.982 [0.232]	-1.326 [0.030]**	0.025 [0.690]	0.165 [0.000]***
Donor's Population	15.420 # [0.000]***	1.32e-06 [0.041]**	0.00001 [0.002]***	5.61e-06 [0.037]**	1.91e-07 [0.000]***	2.40e-07 [0.000]***
lagged Private Lenders' Foreign Claims			-0.013 [0.775]			
lagged ODA Grant Commitment				0.215 [0.005]***		
constant	-232.85 [0.000]***	-156.01 [0.000]***	-212.10 [0.299]	24.956 [0.872]	-5.038 [0.000]***	-10.33 [0.000]***
R^2	0.053	0.112	0.014	-	-	-
F-test / Wald-test (H_0: all $\beta = 0$)	8.38 [0.000]***	3.84 [0.000]***	1.70 [0.039]**	84.27 [0.000]***	1,413.79 [0.000]***	1,642.70 [0.000]***
observation	3,062	2,597	2,144	2,419	3,306	3,256

Note: p-value is reported in [ ] and *, **, *** denotes significance at the 10%, 5%, and 1% level respectively
log transformed.

5. Policy Implications and Way Forward

The empirical results revealed two problems with the IDA's traffic light system.

Firstly, it is providing the opportunities to free ride for the private lenders. This is true even if the traffic light system delivers the correct signal, $\frac{\partial \pi_L}{\partial R} > 0$, to the private lenders, if the private lenders expect that the donor countries would respond more aggressively to the traffic light. Therefore, it is crucial for the donor countries and the multilateral donor agencies to manage the private lenders' expectations.

There are the ex-ante and ex-post approaches. For the ex-ante approach, this could be done by promoting the policy dialogue among the donor communities, the private lenders and the borrower. In terms of each recipient country's context, it would be most efficiently done if the policy dialogue is annually implemented at the timing that the traffic light of the next year is disclosed, especially for the red light countries. Possibly it would be appropriate for the World Bank to promote the policy dialogue at the timing of drafting or revising the Country Assistance Strategy. In terms of broader context, the World Bank/IMF needs to enhance their outreach to other international entities, such as OECD, the Paris Club, the London Club, and so forth where the bilateral aid and the private lending are settled. This would contribute to better management of the private lenders' expectations.

For the ex-post approach, revision to the IDA's policy on non-concessional borrowing should be explored. The current policy burdens the responsibility to avoid non-concessional loans onto the borrower instead of the lenders. Therefore, when it is realized that the country borrows from some lenders in non-concessional terms, then it is the borrower country but not the lenders that are penalized. However, considering that typically the lenders have more bargaining power than the borrower, which is even more true in the developing countries' case, it would not be appropriate to shift

the responsibility only to the borrower country. It is necessary to introduce some counter-measure to punish the lenders' side as well, for instance, excluding the specific non-concessional loans from the international debt settlement negotiation, such as the Paris Club or the London Club, in case that the country falls into sovereign default, which makes the lenders owe the default risk by themselves. It is noteworthy that in some extreme situations there is a possibility that the private lenders may try to extract resources, for instance the bail-out, from the aid donors in collusion with the borrower country by over-lending. Even though this is a risky strategy for the private lenders and the borrower country, it may work under the global/regional financial crises or the debt relief initiatives. In such situations, the close monitoring / surveillance by the multilateral organizations are greatly needed.

Second problem is that the bilateral donor countries are not responding to the traffic light system well enough. Possible reasons behind this are two types of "division of labor". First one is the division of labor between the grant aid and the loan aid agencies in the bilateral donor countries. Under the bilateral aid system, the grant aid agency and the loan aid agency are usually different entity under the control of the different ministries. Typically, the grant aid agency is under control of Ministry of Foreign Affairs, and the loan aid agency is under control of Ministry of Finance. Therefore, each agency provides its own aid modality, grant or loan, to the recipient country, whatever their traffic light may be. This institutional rigidity might be making it difficult for the aid agencies to adjust to the traffic light. Another division of labor is the one among OECD-DAC members. Most of OECD-DAC members provide the grant aid only, and there are only a few countries, such as France, Germany, and Japan, that provide the loan aid in substantial volume. Hence, for the OECD-DAC members who provide the grant aid only, there is no room or reason to adjust their grant aid allocations in response to the recipient countries' traffic light. Whatever the traffic

light is, their aid modality is limited to the grant aid. Since this problem stems from the bilateral aid architecture, it would be difficult for the World Bank/IMF to fix it. However, there seems to be a room for the bilateral aid donors themselves to improve their compliance to the traffic light system. Indeed, some of the bilateral aid donors are making efforts to introduce some innovative financing instruments. For instance, in 2007 French aid agency, AFD, introduced Counter Cyclical Loan. The AFD's Counter Cyclical Loan provides the borrowing country with the option to defer payments when there is a bad external shock, such as export shock, whereby current exports fall below a moving average of past values. In the event of a shock, the borrower country has the opportunity to suspend its capital repayments up to a defined number of suspensions. The grace period of a loan is divided into a fixed initial grace period and a moving grace period, which can be used in the event of a shock. Another example is Japan's Minimal Interest Rate Initiative for Low-Income LDCs, MIRAI. Under this scheme, 0.01% is applied to Japan's ODA loan interest rate. By way of these innovative instruments, some of the bilateral aid donors are able to provide the loan aid without damaging the long-term debt sustainability of the borrower countries, and thus they are responding to the traffic light system correctly. Although these efforts contribute to higher concessionality, predictability, and flexibility of the loan aid, these factors are not reflected in the aid flow data.

There are two issues to be considered for way forward.

Firstly, one of the further refinements of the empirical analysis is to take into considerations the two shocks to the international aid flow. First one is the Gleneagles Summit in 2005. Based on the commitment made then, the bilateral aid donors are supposed to increase aid, especially grant aid, to Africa, which makes the bilateral aid donors to be more selective in providing aid because many of African countries have limited capacity to absorb and manage the increased aid properly. Thus, it is

conceivable that the grant aid has been inclined to flow toward the countries with better institutional capacity, which eventually reduces grant aid to the countries with poorer institutional capacity such as "red-light" countries. As a result, it appears that the bilateral aid donors are not responding to the traffic light appropriately. Second shock is the recent global financial crisis, which reduces the total volume of the international aid flow. Since the current data set includes only up to 2008, the problem is not too serious. But when extending the data set in future, it is necessary to control the effect.

Secondly, there are two other key players who should be included in analyses, the emerging donors and the export credit agencies, both of which have been suspected to be free-riding on the traffic light system. Due to the different reasons of data limitations, both are omitted from the present analyses. For the emerging donors, since they do not owe any responsibility to report their activities to OECD-DAC, there is no systematic data available. For the export credit agencies, they do report to OECD's Credit Reporting System. But due to confidentiality of data, there is not disclosed data for the decomposed to each receiving country. Thus, for both of them only the aggregate flow data are available, which makes impossible to do country-pair analysis. However, as a complement to the present analyses, the aggregate data analyses would be useful. One of the attempt is recently conducted by Mwase (2011), who use aggregated data of Brazil, Russia, India, and China (BRICs)' development financing flows to Low-Income Countries (LICs). It finds that BRICs lend more to resource-rich LICs with weaker institutions, and that their degree of concessionality is negatively correlated with the amount of loans and positively correlated with better institutional indicators. Thus, it suggests that emerging donors are also free riding.

APPENDIX

APPENDIX 1-1: DATA DESCRIPTION AND SOURCES

Bank for International Settlements (BIS), Consolidated Banking Statistics, 2012

CEPII, Geodesic Distances data base, CEPII

IMF, 2012, Directions of Trade Statistics (DOTS), IMF.

OECD, DAC (Development Assistance Committee) database on Aid, 2012.

World Bank, 2012. *Global Development Finance (GDF)*, Washington, D.C.

World Bank, 2012. *World Development Indicators (WDI)*, Washington, D.C.

Variable Name	Variable Description	Data Source
ODA Grant Commitment	DAC countries' grant aid commitment amount (current US$)	OECD-DAC
Export from Donor to Borrower	(million US$)	DOTS
Import by Donor from Borrower	(million US$)	DOTS
Private Lenders' Foreign Claims	Cross-border claims from donor country to borrower country plus foreign offices' local claims (million US$)	BIS
Red Light dummy	Dummy that takes a value of 1 if the country is classified as "red light" by the traffic light system	author's calculation
Real interest rate	Interest rate deflated by GDP	WDI
GNI per capita	GNI per capita (constant 2000 US$)	WDI
GDP growth rate	GDP growth rate (annual %)	WDI
Aid Dependency	Aid % of GNI	WDI
Short-Term Debt	Short-term (maturity less than a year) debt stock % of total external debt stock	GDF
Total Reserves	Total Reserves % of Total External Debt	WDI
Population	Total Population	WDI
Terms of Trade	Net Barter Terms of Trade Index (2000=100)	WDI
Common Official Language	Dummy that takes a value of 1 if two countries share a common official language	CEPII
Colony	Dummy that takes a value of 1 if two countries have ever had a colonial link	CEPII
Distance	Distance between Capital Cities (kilometers)	CEPII

APPENDIX 1-2: SUMMARY STATISTICS

Variable	Mean	Std. Dev.	Min	Max
	Pre-IDA (2000-2004)			
Private Lenders' Foreign Claims	5.86	71.69	-920	1,180.00
ODA Grant Commitment	18.27	56.87	-0.46	1,397.02
Red Light	-	-	-	-
Export from Donor to Borrower	39.10	142.16	0	3,522.60
Import by Donor from Borrower	53.68	366.65	0	15,556.50
Borrowers' Real interest rate	9.84	11.73	-72.56	40.18
Borrowers' GNI per capita	534.66	495.40	80	3,610.00
Borrowers' GDP growth rate	4.50	5.26	-31.30	33.63
Borrowers' Aid Dependency	12.84	10.36	0.40	98.81
Borrowers' Short-Term Debt	8.59	9.21	0	58.27
Borrower's Total Reserves	17.80	15.51	0.01	102.06
Borrower's Population	1.74e+07	2.66e+07	97431	1.39e+08
Borrower's Terms of Trade	98.56	13.46	24.35	143.43
Donors' Real interest rate	3.35	2.50	-0.09	11.62
Donors' GNI per capita	26,253.70	7,776.09	11,670	49,930.00
Donors' GDP growth rate	2.55	1.55	-0.91	5.94
Donor's Population	4.71e+07	6.64e+07	5,165,474	2.93e+08
Common Official Language	0.15	0.36	0	1
Colony	0.05	0.22	0	1
Distance	7,649.82	3,558.55	944.14	17,744.08
	Post-IDA (2005-2008)			
Private Lenders' Foreign Claims	10.33	107.51	1,697	1,783.00
ODA Grant Commitment	37.96	147.41	-2.74	3,830.19
Red Light	0.46	0.50	0	1
Export from Donor to Borrower	88.92	343.83	0	9,016.08
Import by Donor from Borrower	136.58	1,049.14	0	35,652.00
Borrowers' Real interest rate	8.60	8.57	-32.00	37.87
Borrowers' GNI per capita	963.85	895.72	100	5,750
Borrowers' GDP growth rate	5.43	3.99	-7.11	22.70
Borrowers' Aid Dependency	14.07	18.12	0.32	185.94
Borrowers' Short-Term Debt	10.51	11.51	0	65.79
Borrower's Total Reserves	55.11	70.19	0.64	608.47
Borrower's Population	1.98e+07	2.99e+07	100,926	1.58e+08
Borrower's Terms of Trade	111.79	37.90	21.28	251.02
Donors' Real interest rate	1.43	1.97	-2.30	8.18
Donors' GNI per capita	40,814.35	9,971.19	18,060.00	71,520.00
Donors' GDP growth rate	1.20	2.80	-8.35	5.61
Donor's Population	4.90e+07	6.97e+07	5,246,096	3.09e+08
Common Official Language	same as *pre-IDA*			
Colony				
Distance				

APPENDIX 1-3: SAMPLE COUNTRIES

OECD-DAC Members (18 out of 24)

Members	data
Australia	
Austria	
Belgium	
Canada	
Denmark	
Finland	
France	
Germany	
Greece	
Ireland	not included
Italy	
Japan	
Korea	not included
Luxembourg	not included
Netherlands	
New Zealand	not included
Norway	not included
Portugal	
Spain	
Sweden	
Switzerland	
United Kingdom	
United States	
Commission of the European Communities	not included

IDA-only Countries (59 out of 64)

Countries	data
Afghanistan	
Angola	
Bangladesh	
Benin	
Bhutan	not included
Burkina Faso	
Burundi	
Cambodia	
Cameroon	
Cape Verde	
Central Africa Republic	
Chad	
Comoro	
Cote d'Ivore	
Congo, Democratic Republic of	
Republic of Congo	
Djibouti	
East Timor	not included
Eritria	not included
Ethiopia	
Gambia	
Ghana	
Guinea	
Guinea-Bissau	
Guyana	
Haiti	
Honduras	
Kenya	
Kiribati	not included
Kyrgyz Republic	
Lao PDR	
Lesotho	not included
Liberia	
Madagascar	
Malawi	
Maldives	
Mali	
Mauritania	
Moldova	
Mongolia	
Mozambique	
Myanmar	
Nepal	
Nicaragua	
Niger	
Nigeria	
Rwanda	
Samoa	
Senegal	
Sierra Leone	
Somalia	
Solomon Islands	
Saotome & Principe	
Sri Lanka	
Sudan	
Tajikistan	
Tanzania	
Togo	
Tonga	
Uganda	
Vanuatu	
Vietnam	
Yemen	
Zambia	

APPENDIX 1-4: IDA'S TRAFFIC LIGHT SYSTEM <FISCAL YEAR 2013>

Country	GNI per capita	change from previous year
Afghanistan	n.a.	
Angola	4,060	
Bangladesh	770	
Benin	780	Yellow→Green
Bhutan	2,070	
Burkina Faso	570	Red→Yellow
Burundi	250	
Cambodia	830	Yellow→Green
Cameroon	1,210	
Central African Republic	470	
Chad	690	
Comoro	770	
Congo, Democratic Republic of	190	
Congo, Republic of	2,270	
Cote d'Ivore	1,100	Red→Yellow
Djibouti	n.a.	Red→Green
East Timor	n.a.	
Eritrea	430	
Ethiopia	400	
Gambia	610	
Ghana	1,410	
Guinea	440	
Guinea-Bissau	600	
Guyana	n.a.	
Haiti	700	
Honduras	1,970	
Kenya	820	
Kiribati	2,110	
Kosovo	3,520	
Kyrgyz Republic	920	
Lao PDR	1,130	
Lesotho	1,220	
Liberia	240	
Madagascar	430	
Malawi	340	
Maldives	6,530	
Mali	610	
Marshall Islands	3,910	
Mauritania	1,000	
Micronesia	2,900	
Moldova	1,980	
Mozambique	470	
Myanmar	n.a.	
Nepal	540	
Nicaragua	1,170	
Niger	360	Green→Yellow
Nigeria	1,200	
Rwanda	570	
Samoa	3,190	Green→Yellow
Saotome Principe	1,360	
Seirra Leone	340	
Senegal	1,070	
Solomon Islands	1,110	
Somalia	n.a.	
Sudan	1,270	
Tajikistan	870	
Tanzania	540	
Togo	560	
Tonga	3,580	
Tuvalu	5,010	None→Red
Uganda	510	
Vanuatu	2,870	
Yemen	1,070	
Zambia	1,160	

APPENDIX 1-5: OVERVIEW OF IDA'S TRAFFIC LIGHT SYSTEM

The Debt Sustainability Framework (DSF) analyzes both external and public sector debt on the net present value (NPV). To assess debt sustainability, debt burden indicators are compared to indicative thresholds over a 20-year projection period. The DSF classifies countries into one of three policy performance categories (strong, medium, and poor) using the World Bank's Country Policy and Institutional Assessment (CPIA) index, and uses different indicative thresholds for debt burdens depending on the performance category. Thresholds for strong policy performers are highest—indicating that in countries with good policies debt accumulation is less risky.

Debt Burden Thresholds under the DSF

	PV of debt in percent of			Debt service in percent of	
	exports	GDP	Revenue	Exports	Revenue
Weak Policy	100	30	200	15	25
Medium policy	150	40	250	20	30
Strong Policy	200	50	300	25	35

A debt-burden indicator that exceeds its indicative threshold suggests a risk of experiencing some form of debt distress. There are four possible ratings for the risk of debt distress:

low risk *(green light),* when all the debt burden indicators are well below the thresholds;

moderate risk *(yellow light),* when debt burden indicators are below the thresholds, but stress testing indicate that it could be breached in case of external shocks or abrupt policy changes;

high risk *(red light),* when one or more debt burden indicators breach the thresholds; or

in debt distress *(red light),* when the country is already having repayment difficulties.

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CHAPTER 2

NON-MONOTONIC RELATION OF DEBT-DEFAULT PROBABILITY UNDER THE WORLD BANK'S TRAFFIC LIGHT SYSTEM

1. Introduction

In 2004, the World Bank (IDA) introduced the debt sustainability framework (DSF) for low-income countries, so called "traffic light system", based on which IDA determines the grant/loan ratio to be allocated to each recipient country for each fiscal year. For instance, the country receives 100% grant aid, if the country is classified as "red light" (i.e. its debt level is unsustainable), under the traffic light system. The World Bank and IMF have been requesting the other institutions (aid donors, export credit agencies and private lenders) to comply with this system, in order to maintain developing countries' debt sustainability and to avoid another debt crises.

There are some conceivable shortcomings of the traffic light system. One of the serious issues is that the traffic light system is based on the assumption which the relationship between debt and default probability is monotonic. Namely, the higher the debt level, the higher the default probability. However, this assumption may not hold in some cases such that a country makes productive investment by fully utilizing external financing which leads to higher economic growth and eventually reduces its default probability. If this is the case, the current threshold for external borrowing under the traffic light system could be too conservative for some countries. It is critical for the developing countries to grasp to what extent they can borrow and sustainably manage their debt without defaulting, since the developing countries typically experience difficulties to regain access to international credit market and international aid flow after they fall into default.

Therefore I examine the relations between debt indicators and default

probability empirically. My hypothesis is that the current threshold is set at sub-optimal level, which forces the developing countries to bear the costs in the way of foregone development opportunities. Another hypothesis is that the relationship between debt and default probability may not be a monotonic increasing, which is typically assumed. Specifically, the default probability could rather decrease at a certain level of debt, where external debt contribution to growth exceeds its raising effect of default probability (see Figure 1). If that is the case, and if the World Bank's threshold is within the "default probability decreasing zone", it may be possible to conclude that more lending is justified in some countries' cases.

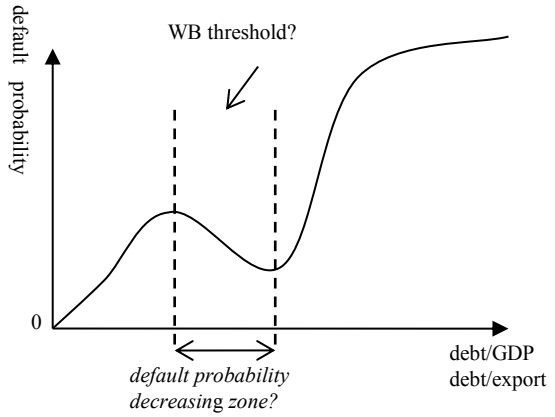


Figure 2-1: Non-Monotonic Relationship between Debt and Default Probability

The main contribution of this paper is to provide the theoretical model and the empirical results of the non-monotonic relationship between debt and default probability by using the panel data of the low-income countries. Since large part of previous (and voluminous) studies on sovereign default focus on emerging economies, this paper contributes to show how sovereign default of low-income countries is different from that of emerging economies.

This paper is organized as follows. The next section reviews the related literature and provides a motivation for the need for research on the monotonicity assumption of the debt-default probability relation. The third section describes the

theoretical model. The fourth section presents the empirical results through the econometric analysis. The last section provides some policy implications and way forward.

2. Literature Review

Although there are abundant studies on sovereign default, the literature of sovereign default can be broadly categorized into two groups; (i) theoretical modeling of sovereign default, and (ii) empirical analyses on determinants of sovereign default.

For the theoretical modeling literatures, there are two approaches; willingness-to-pay approach, and ability-to-pay approach. The willingness-to-pay approach focuses on the case where the borrowing country has the resources to repay but chooses not to because repayment is not optimal choice. Thus, the willingness-to-pay approach depends on the relative costs of defaulting and fulfilling the obligation to repay, as emphasized by Eaton and Gersovitz (1981). The ability-to-pay approach highlights the situation that the borrowing country is unable to meet its repayment obligations due to its debt burden. In this approach, whether the borrowing country is able to repay or not depends on its debt burden, measured, for instance, by the debt stock or the debt service, to its ability to pay, measured, for instance, by GDP, exports or revenues. However, it is hard to distinguish solvency problem from liquidity problem when assessing the country's ability to pay, as described by Roubini (2001). One of the important theoretical works, which is related to ability-to-pay approach, is so-called "debt overhang" and "debt Laffer curve" theory. The literature about the debt overhang and the debt Laffer curve theory dates back to two papers by Krugman (1988) and Sachs (1989). Since then, both the theoretical and empirical works on them have been done. Claessens (1990) shows empirically that there are only a few indebted countries, five out of 29 middle-income countries, are on the wrong side of the debt

Laffer curve. Claessens et al. (1991) find that 6 to 15 out of 35 countries are on the wrong side of the debt Laffer curve, and that the empirical debt Laffer curve is typically flat, which indicates that it would be a mistake to overemphasize the question of whether the debt Laffer curve does eventually bend back. Deshpande (1997) examines the investment experience of 13 severely indebted countries through 1971-1991 and shows that the relationship between the external debt and the investment is negative. Sen et al. (2007) uses several panel data econometric methods to show the existence of a debt overhang which impeded growth in Latin American economies severely and moderately in Asian region. Husain (1997) shows theoretically that a country can be on the wrong side of the debt Laffer curve only if it is on the wrong side of its tax Laffer curve. Agenor and Aizenman (2005) show that lower expected productivity, higher enforcement and verification costs, or higher volatility of productivity shocks, may shift a country to the wrong side of its debt Laffer curve with potentially sizable output and welfare losses. Reinhart, Reinhart, and Rogoff (2012) identifies major debt overhang episodes in the advanced economics since the early 1800s, characterized by public debt to GDP levels exceeding 90% for at least five years, and finds that public debt overhang episodes are associated with growth over one percent lower than during other periods.

For the empirical analyses on determinant of sovereign default, large volume of studies highlight the debt burden as the important factor which triggers the sovereign debt crises. In this perspective, the empirical analyses on determinant of sovereign default share common motivation with studies on a so-called “early warning system”, which use logit or probit regressions typically. Reinhart (2002) finds that a debt crisis is preceded by a currency crisis in about 85% of her sample of 59 countries for 1970-1999, which indicates that variables related to predicting a currency crisis are important to a sovereign debt crisis. Manasse, Roubini and Schimmelpfenning (2003)

estimates a logit model of sovereign debt crises for emerging markets of 1970-2002, and identifies macroeconomic variables reflecting both insolvency, illiquidity and other domestic and external macroeconomic factors that predict a debt crisis episode one year in advance. Their model predict about three quarters of all crises. Some recent literature on financial crisis has proposed on a number of possible non-linear "threshold" effects, by using a non-parametric method. Frankel (2004) use a classic probit and Regression Tree, which is a non-parametric method, to show that a high level of external debt do not necessarily lead to crises on their own but they do significantly raise the probability of crisis if capital inflows is tilted to the short term and is not used to build up reserves. Manasse and Roubini (2005) made an empirical investigation of the set of economic and political conditions that are associated with a likely occurrence of a sovereign debt crisis, based on data of 47 emerging market economies from 1970-2002. They use Binary Recursive Tree model to derive thresholds to classify defaulters and non-defaulters. The approach suggests that it is the particular combination of different types of vulnerability that may lead to default. There are some recent literatures which focus on factors other than debt itself. Reinhart, Rogoff and Savastano (2003) argue that the safe debt thresholds depend on a country's default history, especially serial default record, by using the data set of history of credit events going back to 1820s for over 100 countries. The relevance of institutional quality is widely recognized and being confirmed by the previous studies. Among them, Kraay and Nehru (2004) empirically shows that three factors explain a substantial fraction of the cross-country and time-series variation in the incidence of debt distress: the debt burden, the quality of policies and institutions, and shocks. To the best of the author's knowledge, Minea and Parent (2012) is the sole recent work which casts doubt on monotonicity assumption. Minea and Parent (2012), by using the Panel Smooth Threshold Regression (PSTR) model, shows an endogenously-

estimated threshold around a debt-to-GDP ratio of 115%, above which the negative debt-growth link changes sign. Namely, the economic growth and public debt turns to be positively associated for debt ratio above 115%. Although the previous studies examine in depth the causes which trigger a sovereign default, the underlying common assumption, which is typically implicit, is the monotonic relationship between the default probability and the causes. Thus, higher default probability is derived from the higher level of the causes, whatever they are.

While, there are some recent works on the non-linearity of the debt impact on the economic growth. Pattillo, Poirson and Ricci (2002) assess the non linear impact of external debt on growth using a large panel data set of 93 developing countries over 1969-1998. The findings suggest that the average impact of debt becomes negative at about 160-170% of exports or 35-40% of GDP. The marginal impact of debts starts to be negative at about half of these values. Clements, Bhattacharya, and Nguyen (2003) find that, over a certain threshold, more debt lead to negative rates of growth. Those thresholds are lower than the ones used in the HIPC program (external debt is roughly 115-120% of exports, or 30-37% of GDP.)

My hypothesis is that the above-mentioned non-linear relationship between the debt and the economic growth, together with the linear accumulation of the debt may lead to the non-monotonic relationship between the default probability and the debt. However, none of the previous studies shed a light on this potential non-monotonicity, which is this paper's contribution.

3. Theoretical Model

In order to derive the non-monotonic relationship between debt stock and default probability, the theoretical model is constructed by modifying the Debt Overhang / Debt Laffer Curve model of Obstfeld and Rogoff (1994).

3.1. Debt Overhang / Debt Laffer Curve Model

Consider two-period problem. On the 1st period, the debtor country has an inherited debt stock of face value of D, owing to the foreign creditors, that will come due on the 2nd period. The debtor country's income is Y_1 in the 1st period and $A \cdot F_2(K_2) \cdot F_0(\delta D)$ in the 2nd period, where A is the productivity shock with mean $E[A] = 1$ and distributed over $[\underline{A}, \overline{A}]$ with probability density function $f(A)$. K_2 is the capital invested in the 1st period, and $F_2(K_2)$ is its realized return in the 2nd period, which is augmented by $F_0(\delta D)$, interpreted as the long-term investment such as large-scale infrastructure, which is invested before the 1st period. It is assumed that only some fraction, δ, of the inherited debt was accumulated as the productive capital, due to institutional / political constraints. Thus, δ is interpreted as the efficiency measure of public investment. I assume the debtor country is risk-neutral with expected utility function; $U = C_1 + E[C_2]$, which is linear in consumption with no subjective discount factor.

The creditors penalize the debtor country in the amount of $\eta\, AF_2(K_2)F_0(\delta D)$, in case of default. Eliminating consumption levels by using constraints,

$$C_1 = Y_1 - K_2, \qquad C_2 = AF_2(K_2)F_0(\delta D) - \min[\eta AF_2(K_2)F_0(\delta D),\ D]$$

Then the debtor country's maximization problems is

$$\max_{K_2} U(K_2) = Y_1 - K_2 + E\left[AF_2(K_2)F_0(\delta D) - \min[\eta\, AF_2(K_2)F_0(\delta D), D]\right]$$

$$= Y_1 - K_2 + F_2(K_2)F_0(\delta D) - V(D, K_2) \qquad (1)$$

where $V(D, K_2)$ is the repayment which creditors actually expect to receive on the 2nd period. (And this sum is the debt's market value.) Since the debtor country choose to default for realization of A such that $\eta\, AF_2(K_2)F_0(\delta D) < D$, that is when $A < D/\{\eta F_2(K_2)F_0(\delta D)\}$,

$$V(D,K_2) = \eta F_2(K_2)F_0(\delta D)\left[\int_{\underline{A}}^{\frac{D}{\eta F_2(K_2)F_0(\delta D)}} Af(A)dA\right] + D\left[\int_{\frac{D}{\eta F_2(K_2)F_0(\delta D)}}^{\overline{A}} f(A)dA\right] \quad (2)$$

The first term in the right-hand side of equation (2) captures payment in case of default and the second term captures payment in case of non-default.

To see how the increase of the inherited debt stock affect the debtor country's optimal investment, plugging (2) into (1), the following first order and second conditions are derived.

$$F_2'(K_2)F_0(\delta D)\left[1-\eta\int_{\underline{A}}^{\frac{D}{\eta F_2(K_2)F_0(\delta D)}} Af(A)dA\right] = 1 \quad (3)$$

$$U''(K_2) = F_2''(K_2)F_0(\delta D)\left[1-\eta\int_{\underline{A}}^{\frac{D}{\eta F_2(K_2)F_0(\delta D)}} Af(A)dA\right] + \frac{D^2F_2'(K_2)f\left(\frac{D}{\eta F_2(K_2)F_0(\delta D)}\right)}{\eta\{F_2(K_2)\}^3F_0(\delta D)} < 0 \quad (4)$$

The first order condition (3) states that the debtor country will invest up to a point where the expected marginal product of investment, net of expected additional penalty payment to creditors, equals the current consumption cost of investing (that is, 1). Thus, (3) defines the optimal investment $K_2(D)$. By implicit function theorem,

$$\frac{dK_2}{dD} = K_2'(D)$$

$$= -\frac{F_2'(K_2)F_0'(\delta D)\delta\left[1-\eta\int_{\underline{A}}^{\frac{D}{\eta F_2(K_2)F_0(\delta D)}} Af(A)dA\right] + \frac{DF_2'(K_2)f\left(\frac{D}{\eta F_2(K_2)F_0(\delta D)}\right)\{\delta DF_0'(\delta D)-F_0(\delta D)\}}{\eta\{F_2(K_2)\}^2\{F_0(\delta D)\}^2}}{U''(K_2)} \quad (5)$$

Since the denominator is negative from (4), the sign of $K_2'(D)$ is that of the

numerator. Thus,

$$K_2'(D) < 0 \quad \text{if } D > \frac{F_0\prime(\delta D)\delta\eta\{F_2(K_2)\}^2 F_0(\delta D)}{F_2\prime(K_2) f\left(\frac{D}{\eta F_2(K_2) F_0(\delta D)}\right)\{\delta D F_0\prime(\delta D) - F_0(\delta D)\}} \equiv D_3 \tag{6}$$

$$K_2'(D) > 0 \quad \text{if } D < \frac{F_0\prime(\delta D)\delta\eta\{F_2(K_2)\}^2 F_0(\delta D)}{F_2\prime(K_2) f\left(\frac{D}{\eta F_2(K_2) F_0(\delta D)}\right)\{\delta D F_0\prime(\delta D) - F_0(\delta D)\}} \equiv D_3 \tag{7}$$

Especially, note that if $\delta D F_0{}'(\delta D) - F_0(\delta D) > 0$, then $K_2'(D)$ is always positive. In other words, as long as the production function from the inherited debt is elastic, higher the inherited debt level is, higher the investment in the first period is made. This is different from the traditional *debt overhang* effect, which represents the situation which the inherited debt stock always has a negative effect on debtor country's investment.

To see how the inherited debt stock affect the debtor country's default probability, $\pi(D)$, differentiate default probability with respect to D, taking account of dependent of K_2 on D.

$$\begin{aligned} &\frac{d}{dD}\left[1 - \int_{\frac{D}{\eta F_2(K_2) F_0(\delta D)}}^{\overline{A}} f(A)dA\right] \\ &= \pi'(D) \\ &= \frac{f\left(\frac{D}{\eta F_2(K_2) F_0(\delta D)}\right)}{\eta}\left[\frac{F_2(K_2)F_0(\delta D) - D\{K_2'(D)F_0(\delta D) - \delta F_2(K_2)\}}{\{F_2(K_2)\}^2\{F_0(\delta D)\}^2}\right] \end{aligned} \tag{8}$$

Since all the term except for the numerator of the second term is positive, the sign of $\pi'(D)$ is that of the numerator. Thus,

$$\pi'(D) < 0 \quad \text{if } D > \frac{F_2(K_2)F_0(\delta D)}{K_2'(D)F_0(\delta D) - \delta F_2(K_2)} \equiv D_1, \tag{9}$$

$$\text{where } K_2'(D) > 0 \ \& \ \frac{K_2'(D)}{\delta} > \frac{F_2(K_2)}{F_0(\delta D)}$$

$$\pi'(D) > 0 \quad \text{if } D < \frac{F_2(K_2)F_0(\delta D)}{K_2'(D)F_0(\delta D) - \delta F_2(K_2)} \equiv D_1, \tag{10}$$

$$\text{where } \frac{K_2'(D)}{\delta} > \frac{F_2(K_2)}{F_0(\delta D)} \text{ or } K_2'(D) < 0$$

Define the debt stock D_2 is such that $\frac{K_2'(D)}{\delta} = \frac{F_2(K_2)}{F_0(\delta D)}$. From (5),

$$D_2 \equiv \frac{\delta\left\{F_0{}'(\delta D)-\frac{F_2(K_2)}{F_0(\delta D)}\right\}}{\frac{f\left(\frac{D}{\eta F_2(K_2)F_0(\delta D)}\right)F_2'(K_2)\{F_0(\delta D)-\delta D F_0{}'(\delta D)\}}{\eta\{F_2(K_2)\}^2 F_0(\delta D)}} \quad (11)$$

Combining (6), (7), (9), (10), and (11), the condition for decreasing default probability is following. The debt stock effect on the default probability is illustrated in Figure 2-2.

$$\pi'(D) < 0 \text{ if } \frac{F_2(K_2)F_0(\delta D)}{K_2'(D)F_0(\delta D)-\delta F_2(K_2)} < D < \frac{F_0{}'(\delta D)\delta\eta\{F_2(K_2)\}^2 F_0(\delta D)}{F_2{}'(K_2)f\left(\frac{D}{\eta F_2(K_2)F_0(\delta D)}\right)\{\delta D F_0{}'(\delta D)-F_0(\delta D)\}} \quad (12)$$

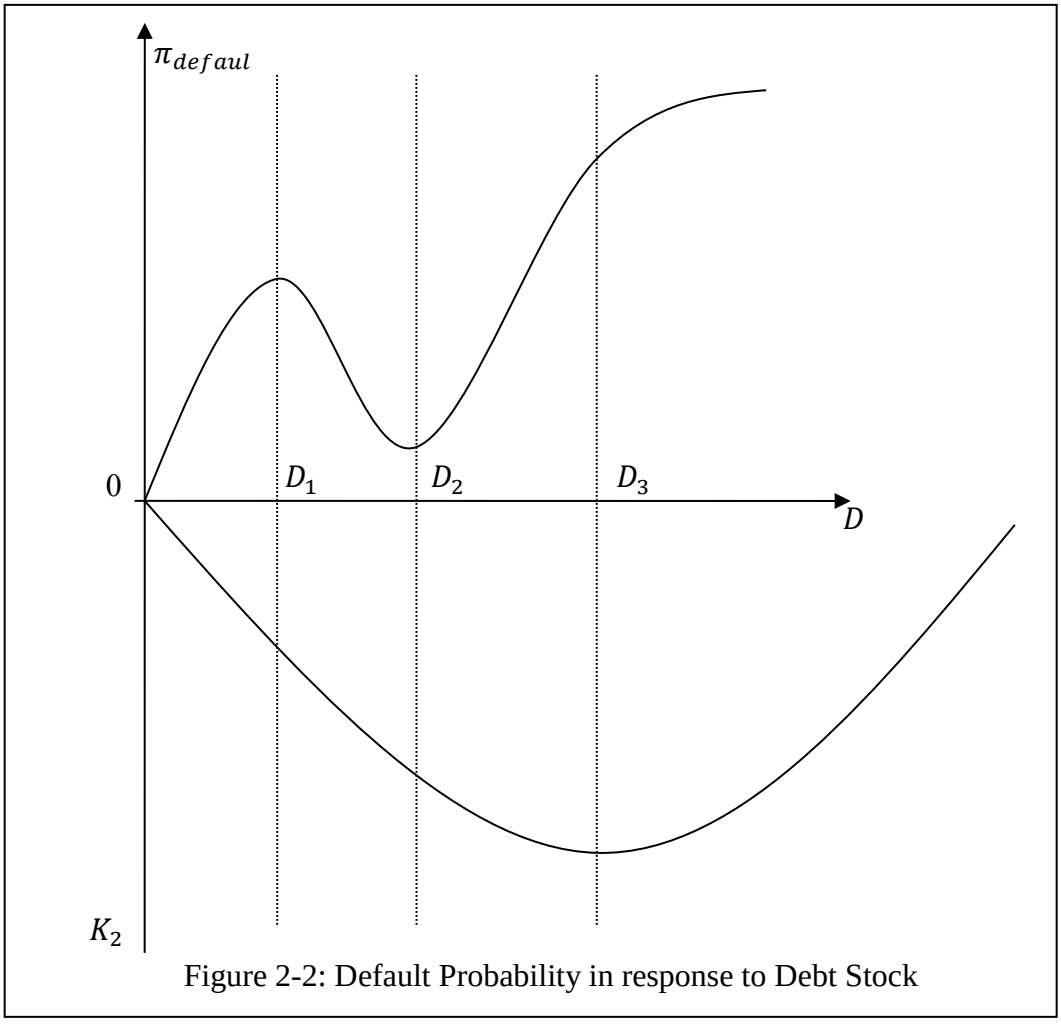


Figure 2-2: Default Probability in response to Debt Stock

3.2. Numerical Analysis

Before moving on to the empirical analysis, the numerical analysis is conducted on the specific functional forms and the parameter values.

<Case 1> Uniform Distribution

The probability distribution of the productivity shock, A, is assumed to uniformly distributed over $[\underline{A},\ \overline{A}]$ with mean $E[A] = 1$, for simplicity. Therefore, $\underline{A} = 2 - \overline{A}$. (Please see Table 2-1 for the functional forms and Table 2-2 for the parameter values.)

Table 2-1: Functional Forms for Uniform Distribution Case

$F_0(\delta D)$	$\beta_0 + \beta_1(\delta D) + \beta_2(\delta D)^2$
$F_2(K_2(D))$	$(K_2)^{\alpha}$

Table 2-2: Parameter Values for Uniform Distribution Case

β_0	1
β_1	2
β_2	-1
α	0.8
δ	0.5
η	0.9
$\overline{A}$	2
$\underline{A}$	0

Then, the default probability, $\pi(D)$, takes the following form;

$$\pi(D) = \int_{\underline{A}}^{\frac{D}{\eta F_2(K_2)F_0(\delta D)}} f(A)dA = \frac{\overline{A}-2}{2\overline{A}-2} + \frac{1}{2\overline{A}-2}\cdot\frac{D}{\eta F_2(K_2)F_0(\delta D)} \quad (13)$$

The first term of (13) is negative if $\overline{A} < 2$, positive if $\overline{A} > 2$, and zero if $\overline{A} = 2$. Thus, it takes larger values as the range of $[\underline{A},\ \overline{A}]$ widens. At a first glance, it may seem weird, since the default probability can take positive values even if there is

no debt (i.e. $\pi(0) > 0$), which implies that the country can default even if it owes no debt. However, it has the reasonable economic interpretation. The first term of (13) is the "intrinsic" default probability with which the country is burdened, due to its range of productivity shock (i.e. volatility of productivity.). If the country has large volatility of productivity (i.e. if $\overline{A} > 2$ or $\underline{A} < 0$), then it potentially bears positive default probability even if it owes no debt. This makes sense, because if the productivity shock can be negative (i.e. $\underline{A} < 0$), the country must have potential default probability for any incremental debt. In the numerical analysis, $\overline{A} = 2$ is assumed for simplicity. Thus, there is no negative productivity shock.

Both $F_0(\delta D)$ and $F_2(K_2)$ take the concave functional forms for the range of $[0,\ 2]$. Since $F_0(\delta D)$ is assumed to augment $F_2(K_2)$, it has the intercept of 1.

The optimal investment K_2 depends on the inherited debt stock, D, and $K_2(D)$ is defined such that (3) is satisfied. For the uniform distribution and the given functional forms, (3) becomes the following condition, which cannot be explicitly solved;

$$F_2'(K_2)F_0(\delta D)\left[1-\eta\int_{\underline{A}}^{\frac{D}{\eta F_2(K_2)F_0(\delta D)}} Af(A)dA\right]=1$$

$$\Rightarrow (K_2)^{2\alpha}-\frac{1}{\alpha\cdot\{\beta_0+\beta_1(\delta D)+\beta_2(\delta D)^2\}}(K_2)^{\alpha+1} -\frac{D^2}{4\eta\cdot\{\beta_0+\beta_1(\delta D)+\beta_2(\delta D)^2\}^2} \tag{14}$$

The result of the numerical analysis is plotted in Figure 2-3.

As can be seen, the default probability increases up to around 12.5% during first stage, D_1, then slowly but steadily declines to around 7.5% during the second stage, D_2, and finally increases again as the inherited debt stock reaches to the value of 2 during the third stage, D_3.

The interpretation of the non-monotonicity is as follows. For the first stage, the output grows slower relative to the inherited debt stock accumulation, which makes the default probability increase. For the second stage, the investment becomes productive enough to exceed the default probability increasing effect caused by the inherited debt stock accumulation, which makes the default probability makes decrease. For the third stage, the investment gets less productive due to the concavity of F_0 as well as F_2, then the debt accumulation grows faster, which makes the default probability increase again.

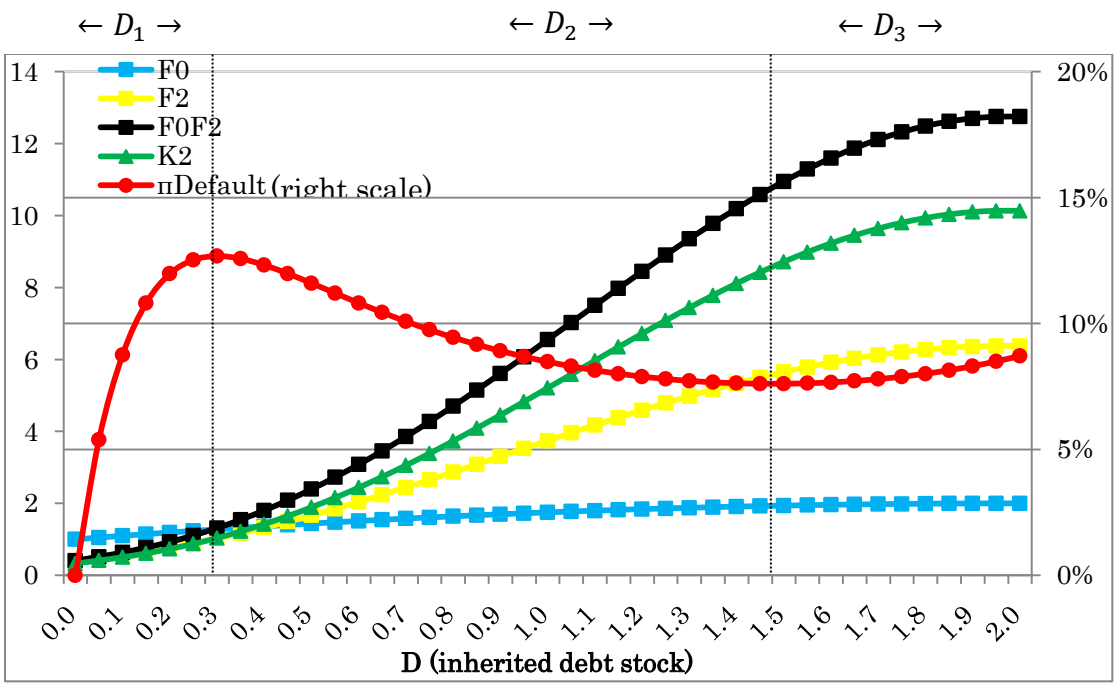


Figure 2-3: Numerical Analysis for Uniform Distribution

<Case 2> Exponential Distribution

Exponential Distribution is assumed for the probability distribution of the productivity shock, A, with mean $E[A] = 1$, for simplicity. (Please see Table 2-3 for the functional forms and Table 2-4 for the parameter values.)

Table 2-3: Functional Forms for Exponential Distribution Case

$F_0(\delta D)$	$\beta_0 + (\delta D)^{\gamma}$
$F_2(K_2(D))$	$(K_2)^{\alpha}$

Table 2-4: Parameter Values for Exponential Distribution Case

β_0	1
γ	0.9
α	0.8
δ	0.5
η	0.9

Then, the default probability, $\pi(D)$, takes the following form;

$$\pi(D) = \int_0^{\frac{D}{\eta F_2(K_2)F_0(\delta D)}} f(A)dA = 1 - e^{-\left(\frac{D}{\eta F_2(K_2)F_0(\delta D)}\right)} \quad (15)$$

Both $F_0(\delta D)$ and $F_2(K_2)$ take the concave functional forms. Since $F_0(\delta D)$ is assumed to augment $F_2(K_2)$, it has the intercept of 1.

The optimal investment K_2 depends on the inherited debt stock, D, and $K_2(D)$ is defined such that (3) is satisfied;

$$\Rightarrow F_2'(K_2)F_0(\delta D)\left[1 - \eta\left\{1 - e^{-\left(\frac{D}{\eta F_2(K_2)F_0(\delta D)}\right)} \cdot \left(1 + \frac{D}{\eta F_2(K_2)F_0(\delta D)}\right)\right\}\right] = 1 \quad (16)$$

The result of the numerical analysis is plotted in Figure 2-4.

As can be seen, the default probability increases up to more than 70% during first stage, D_1, then slowly but steadily declines to around 50% during the second stage, D_2, and finally increases again during the third stage, D_3.

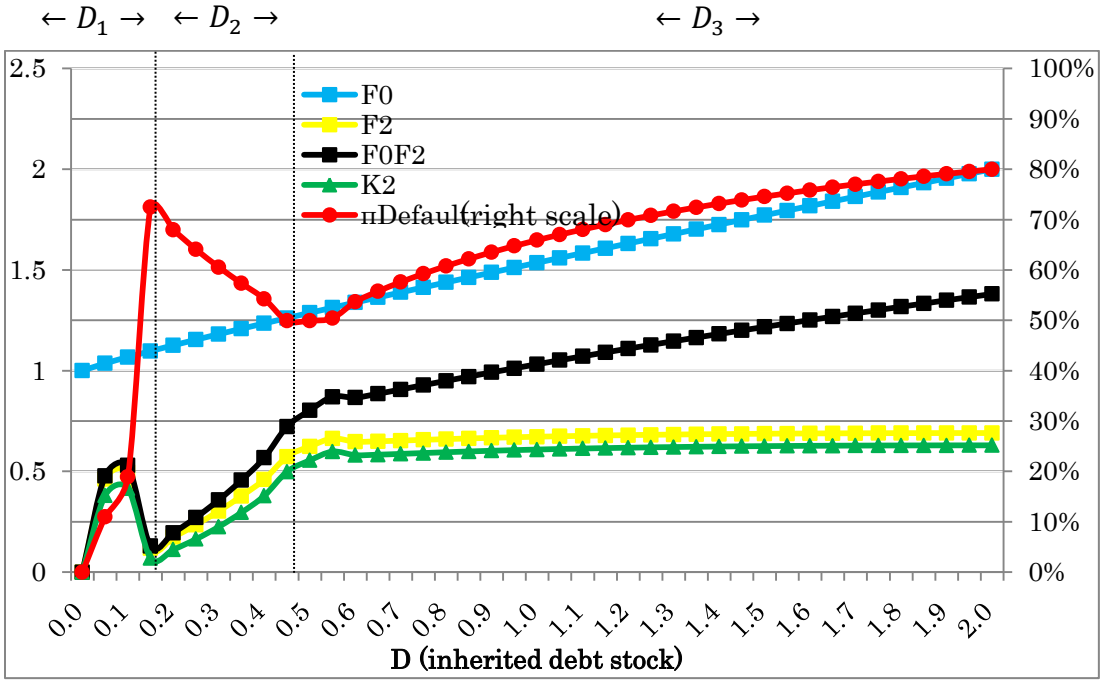


Figure 2-4: Numerical Analysis for Exponential Distribution

<Case 3> Log-Normal Distribution

Log-Normal Distribution is assumed for the probability distribution of the productivity shock, A, with mean $E[A] = 1$, for simplicity. (Please see Table 2-5 for the functional forms and Table 2-6 for the parameter values.)

Table 2-5: Functional Forms for Log-Normal Distribution Case

$F_0(\delta D)$	$\beta_0 + \beta_1 ln(\delta D)$
$F_2(K_2(D))$	$(K_2)^{\alpha}$

Table 2-6: Parameter Values for Log-Normal Distribution Case

β_0	2
β_1	0.5
α	0.5
δ	0.8
η	0.2
μ	-0.5
σ	1

Then, the default probability, $\pi(D)$, takes the following form. Although there is no closed form solution, it can be expressed with error function, erf.

$$\pi(D) = \int_0^{\frac{D}{\eta F_2(K_2)F_0(\delta D)}} f(A)dA = \frac{1}{2}\cdot\left[1 + erf\left(\frac{ln\left(\frac{D}{\eta F_2(K_2)F_0(\delta D)}\right) - \mu}{\sqrt{2\sigma^2}}\right)\right] \quad (17)$$

Both $F_0(\delta D)$ and $F_2(K_2)$ are takes the concave functional forms.

The optimal investment K_2 depends on the inherited debt stock, D, and $K_2(D)$ is defined such that (3) is satisfied;

$$\Rightarrow \int_0^{\frac{D}{\eta F_2(K_2)F_0(\delta D)}} Af(A)dA = \int_0^{\frac{D}{\eta F_2(K_2)F_0(\delta D)}} A\cdot\frac{1}{A\sqrt{2\pi\sigma^2}}\cdot e^{-\left(\frac{(lnA-\mu)^2}{2\sigma^2}\right)}dA$$

$$\Rightarrow = \frac{1}{\sqrt{2\pi\sigma^2}} \int_0^{\frac{D}{\eta F_2(K_2)F_0(\delta D)}} e^{-\left(\frac{(lnA-\mu)^2}{2\sigma^2}\right)}dA$$

Let lnA be x. Then, $\frac{dx}{dA} = \frac{1}{A}$, so $dA = A\cdot dx$ and $A = e^x$. Then,

$$\Rightarrow \int_0^{\frac{D}{\eta F_2(K_2)F_0(\delta D)}} Af(A)dA = \frac{1}{\sqrt{2\pi\sigma^2}} \int_0^{\frac{D}{\eta F_2(K_2)F_0(\delta D)}} e^{-\left(\frac{(x-\mu)^2}{2\sigma^2}\right)}\cdot A\cdot dx$$

$$\Rightarrow = \frac{1}{2}\cdot\left[1 + erf\left(\frac{ln\left(\frac{D}{\eta F_2(K_2)F_0(\delta D)}\right) - (\mu+\sigma^2)}{\sqrt{2\sigma^2}}\right)\right]$$

$$\Rightarrow F_2'(K_2)F_0(\delta D)\left[1 - \eta\cdot\frac{1}{2}\cdot\left[1 + erf\left(\frac{ln\left(\frac{D}{\eta F_2(K_2)F_0(\delta D)}\right) - (\mu+\sigma^2)}{\sqrt{2\sigma^2}}\right)\right]\right] \quad (18)$$

$$= 1$$

The result of the numerical analysis is plotted in Figure 2-5.

As can be seen, the default probability increases up to 97% during first stage, D_1, then suddenly declines to around 88% during the second stage, D_2, and finally and steadily increases again during the third stage, D_3.

Both the exponential distribution and the log-normal distribution exhibits higher default probability than the uniform distribution. This is because both the exponential distribution and the log-normal distribution is skewed to left tail, which indicates that smaller productivity shock is more likely to happen. Therefore, both probability distributions' case exhibits higher default probability.

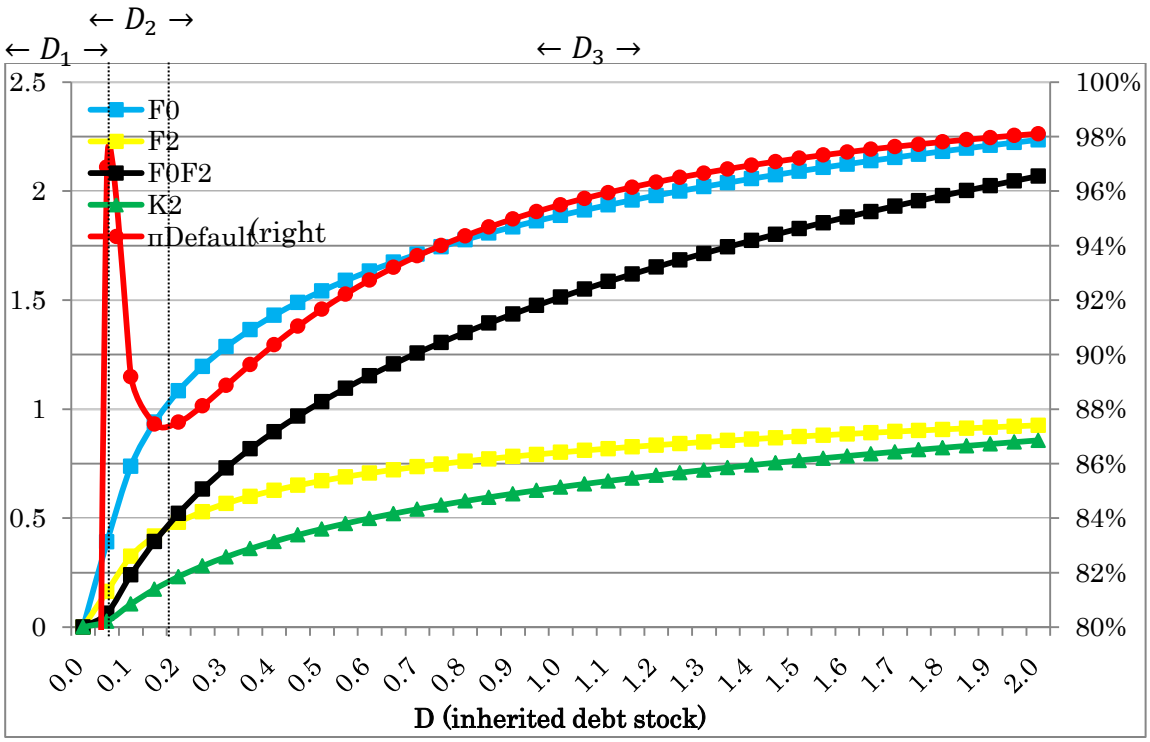


Figure 2-5: Numerical Analysis for Log-Normal Distribution

4. Empirical Analysis

Following the standard sovereign default study, the Panel Logit / Probit Model are applied. The data set covers 129 of low-income / middle income countries

(definition based on the World Bank) for the period 1970-2011.

The dependent variable is the categorical variable of default or not. The definition of default is the central to the sovereign default study, but it varies in the preceding literatures. They can be roughly categorized into three approaches. (See Pescatori and Sy (2007) for a overview of definition of default.) First approach is to list up the case studies and anecdotal episodes of default events in history. (See Reinhart (2010) for a comprehensive and detailed overview of default in country-by-country samples.) Second approach applies a quantitative definition. For instance, Detragiache and Spilimbergo (2001) define a country to be in a debt crisis if the country has arrears on external obligations toward commercial creditors in excess of 5 percent of commercial debt outstanding or has a rescheduling or restructuring agreement with commercial creditors. Sy (2004) defines debt crises as sovereign bond distress events, and suggests that sovereign bonds are distressed securities when bond spreads are trading 1,000 bps or more above U.S. Treasury securities, since the author argues that in practice, the 1,000 bps mark for spreads is often considered a psychological barrier by market participants. Third approach uses the rating defined by the rating agencies. All in all, there is no widely accepted consensus on how to define a sovereign default.

The standard three approaches do not fit to the low income countries' default. First approach does not work well, since the low income countries typically experience serial defaults, and some of them experience even "overlapping" defaults, in which the countries fall into another default before they settle the previous default with the creditors, which makes it difficult to separate one default episode from another. Second approach may causes selection bias problem. Third approach is not applicable to the low income countries, since only one-quarter of the low income countries have been rated by at least one of the three major agencies (Standard &

Poor's, Moody's and Fitch).

IMF and the World Bank (2012) focuses on three indicators related to exceptional external financing to signal whether a country is experiencing debt distress: (i) the accumulation of arrears on public and publicly guaranteed external debt in excess of five percent of the public and publicly guaranteed external debt stock outstanding; (ii) a rescheduling of obligations due to Paris Club creditors; or (iii) the disbursement by the IMF of General Resources Account resources exceeding 50 percent of IMF quota. A debt distress episode is defined as a period lasting three or more years in which at least one distress signal is observed. The World Bank and IMF's definition is a mixture of the first and second approach mentioned above, and thus bears the same problems as above.

Here I use the following two simple definitions;

(i) *default* is defined as the situation if the arrears are increasing from the previous year. Total number of events is 806 for Low-Income Countries.

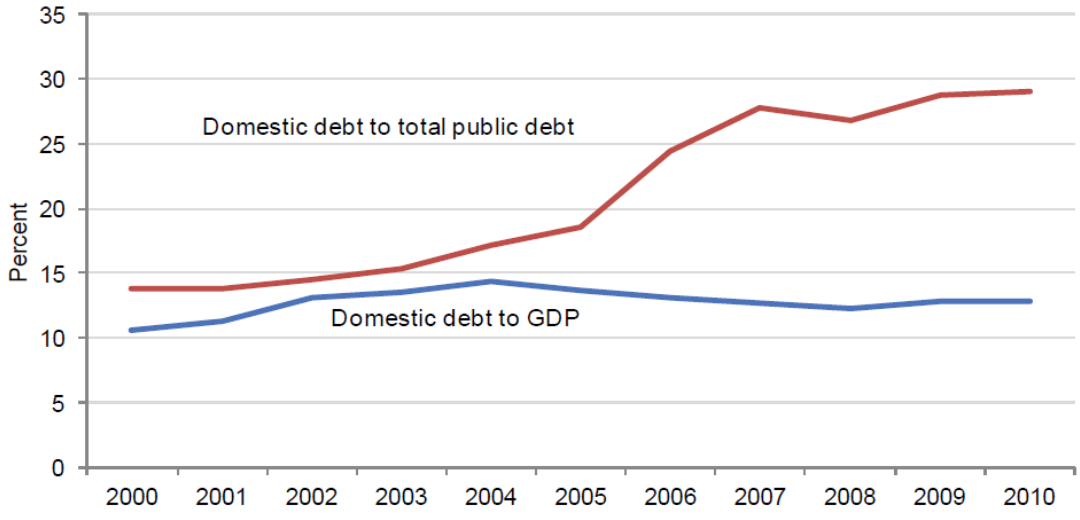
(ii) *default(serial)* is defined as the situation if the arrears are increasing more than two consecutive years. This eliminates the possibility of "technical delay" of repayment. The number of default events is 538 for Low-Income Countries.

Thus, the definition applied here is simplistic but can capture the sovereign defaults in a broad sense. The weakness of my definition is supposed to be its sensitivity to the international donors' assistance from abroad, since the non-default (i.e. no increases of the arrears) might be simply the result of the external assistance inflow. Thus, the external assistance variable is important control variable. It is necessary to bear this point in mind when interpreting the derived results.

For the explanatory variables, the variable of interest is the debt stock variable. Here the external debt stock (% of GDP) and (% of export) are applied, since these are the variables used in the World Bank's Debt Sustainability Framework. It

should be noted that the external debt stock is transformed into the net present value term, as the World Bank's Debt Sustainability Framework use the net present value term. But since loan-by-loan data is not publicly available, the net present value calculation is only an approximation. In order to capture the non-monotonicity, the original, quadratic, and cubic terms of the external debt stock enters the estimated equation, with the positive, negative, and positive expected sign, respectively.

When considering the debt stock, the domestic debt, rather than the external debt, is becoming an issue of significance. IMF and the World Bank (2012) pointed out that since 2005 the share of domestic public debt in total public debt has increased from 19 percent to 29 percent on average across all the low income countries. However, this derives largely from external debt relief and the domestic debt among total public debt rises relatively. In fact, domestic debt as a ratio to GDP has remained flat at the level of 10-15% level on average since 2000. (See Figure 2-6.) Therefore, in the following empirical analyses, the domestic debt is not incorporated.



Source: IMF and the World Bank (2012)

Figure 2-6: Domestic Debt to Total Public Debt and to GDP

Other control variables are taken from the standard sovereign default studies. The debt service ratio is supposed to capture the flow aspect of the country's debt burden, while the variable of interest, the external debt stock, captures the stock aspect. The foreign direct investment and workers' remittance are included to reflect the diverse financial inflow to the low income countries, and to control their impact on sovereign default. The GDP growth, the inflation rate, the total reserves are supposed to represent the country's economic fundamentals. The government final consumption expenditure ratio, which includes all government current expenditures, is included to reflect the government's efficiency of public expenditure. Thus, it would be a proxy of δ, which is the efficiency measure of public investment described in the theoretical model. Thus, the economic downturn or turmoil would be captured by these variables. The previous studies emphasize that it is not only the total volume of the debt but its composition, especially the short-term debt ratio, is critical to the defaults. The dummy variable of the low income countries is included to recognize whether the sovereign defaults of the low income countries are different from emerging economies. In addition, the official creditors' commitment is included, in order to control their, presumably negative, impact on the default probability.

4.1. Main Results

The results are summarized in Table 2-7.

Table 2-7: Estimation Results for External Debt Stock (% of GDP)

X_{it} \ Y_{it}	Logit				Probit	
	Fixed Effect Model		Random Effect Model		Random Effect Model	
	default	*default (serial)*	*default*	*default (serial)*	*default*	*default (serial)*
External Debt Stock (% of GDP)	0.056 [0.000]***	0.063 [0.000]***	0.060 [0.000]***	0.062 [0.000]***	0.036 [0.000]***	0.039 [0.000]***
External Debt Stock (quadratic) (% of GDP)	-0.0004 [0.002]***	-0.0004 [0.001]***	-0.0005 [0.000]***	-0.0005 [0.000]***	-0.0003 [0.000]***	-0.0003 [0.000]***
External Debt Stock (cubic) (% of GDP)	1.18e-06 [0.017]**	1.09e-6 [0.012]**	1.31e-06 [0.006]***	1.12e-06 [0.005]***	7.77e-07 [0.005]***	6.96e-07 [0.002]***
Debt Service Ratio (% of export)	-0.001 [0.829]	-0.003 [0.684]	-0.006 [0.286]	-0.009 [0.148]	-0.003 [0.323]	-0.005 [0.217]
GDP growth (annual %)	-0.025 [0.0014]**	-0.038 [0.000]***	-0.028 [0.006]***	-0.040 [0.000]***	-0.016 [0.005]***	-0.024 [0.000]***
Inflation Rate, GDP deflator (annual %)	0.001 [0.081]*	0.0003 [0.224]	0.001 [0.080]*	0.0003 [0.212]	0.001 [0.056]*	0.0002 [0.180]
Short-term debt (% of total external debt)	0.017 [0.010]**	0.031 [0.000]***	0.010 [0.085]*	0.019 [0.003]***	0.006 [0.084]*	0.012 [0.002]***
Total Reserve (% of total external debt)	-0.0003 [0.676]	-0.0003 [0.831]	-0.001 [0.350]	-0.003 [0.173]	-0.001 [0.349]	-0.0003 [0.547]
LIC (1=Low Income Countries)	-0.544 [0.008]**	-0.715 [0.001]***	-0.139 [0.412]	-0.214 [0.237]	-0.078 [0.444]	-0.115 [0.279]
Official Creditors' Commitment (million USD)	-0.0002 [0.019]**	-0.0002 [0.109]	-0.0003 [0.000]***	-0.0003 [0.001]***	-0.0001 [0.000]***	-0.0002 [0.000]***
Foreign Direct Investment (% of GDP)	-0.043 [0.003]***	-0.065 [0.000]***	-0.044 [0.001]***	-0.059 [0.000]***	-0.027 [0.001]***	-0.036 [0.000]***
Remittance (% of GDP)	-0.007 [0.628]	-0.048 [0.008]***	-0.008 [0.404]	-0.029 [0.022]**	-0.005 [0.396]	-0.017 [0.020]**
Government Expenditure (% of GDP)	-0.006 [0.670]	-0.019 [0.186]	0.005 [0.676]	-0.003 [0.809]	0.003 [0.650]	-0.002 [0.760]
observation	2,324	2,161	2,516	2,507	2,516	2,507
Hausman test (H_0: Random-Effect Model is correct.)	19.68 [0.073]*	42.57 [0.000]***	-	-	-	-
Default Probability Decreasing Zone	97 – 164%	97 – 195%	89 – 170%	99 – 185%	90 –173%	99 – 187%

Note: p-value is reported in [ ] and *, **, *** denotes significance at the 10%, 5%, and 1% level.

The findings are basically common across the models, Logit, Probit. The main finding is the stock-related variables, *External Debt Stock (% of GDP)*, are statistically significant with expected signs for each in original, quadratic and cubic terms. The relationship between estimated default probability and external debt stock is plotted in Figure 2-7 through Figure 2-12 which is implying that there is a non-monotonic relationship between debt stock and default probability. Adding the quadratic and the cubic terms of the external debt stock is somewhat arbitrary. However, both the Akaike Information Criteria (AIC) and the Schwarz Bayesian Information Criteria (SBIC) return the better scores for adding the quadratic and the cubic than without adding any or adding higher terms.

Among the other control variables, the GDP growth enters with negative sign with statistical significance at all models. Thus, higher economic growth seems to

reduce the default probability. On the other hand, the short-term debt has the positive sign with statistical significance in all models. The dummy variable of the low income countries is statistically significant only for the fixed effect model, but it has the negative sign in all models explored, which indicates that the low income countries have relative low default probability. The official creditors' commitment has the expected negative sign, and is statistically significant in most of the models. Foreign direct investment enters with negative signs with statistical significance in all models, and the workers' remittance also presents similar results. Thus, both the foreign direct investment and remittance inflow reduces the default probability. The government expenditure does not have statistical significance in any models, therefore it would be necessary to explore other variables to be a proxy for public expenditure efficiency.

The estimated default probability decreasing zone is located at the range of 89-195% of the external debt stock % of GDP. This seems significantly high, comparing to the threshold of the World Bank's Debt Sustainability Framework, such as 40% for the Medium strength policy country.

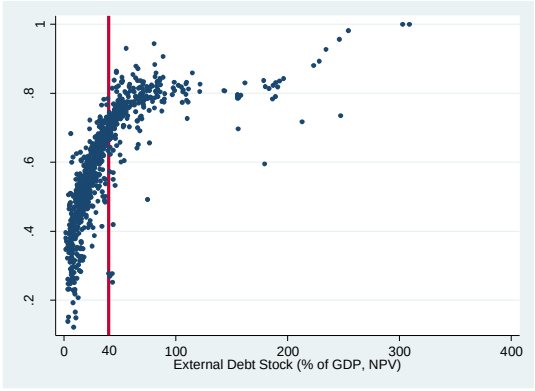


Figure 2-7: External Debt % of GDP
(Dependent Variable: *default*
Model: Logit Fixed-Effect)

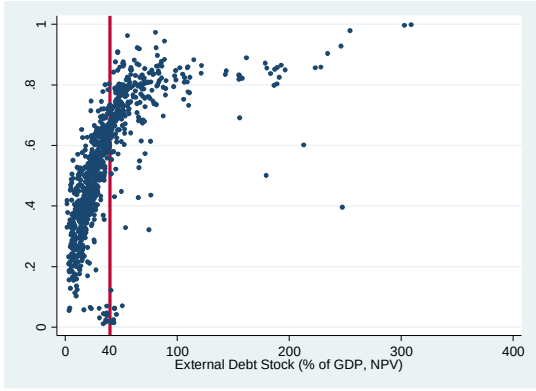


Figure 2-8: External Debt % of GDP
(Dependent Variable: *default(serial)*
Model: Logit Fixed-Effect)

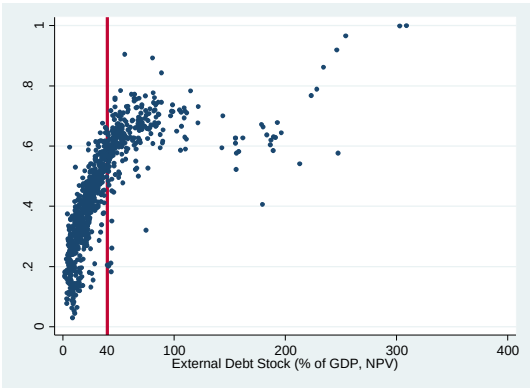


Figure 2-9: External Debt % of GDP
(Dependent Variable: *default*
Model: Logit Random-Effect)

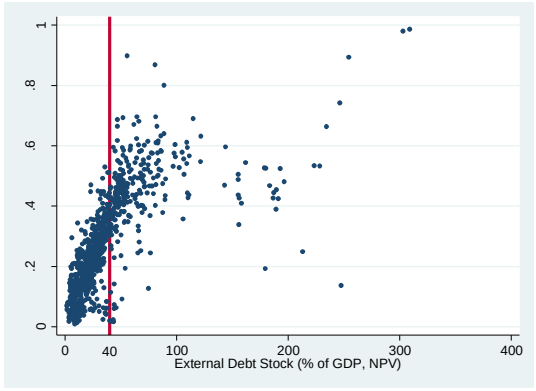


Figure 2-10: External Debt % of GDP
(Dependent Variable: *default(serial)*
Model: Logit Random-Effect)

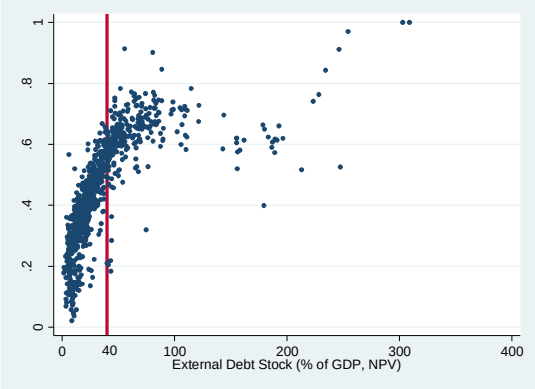


Figure 2-11: External Debt % of GDP
(Dependent Variable: *default*
Model: Probit Random-Effect)

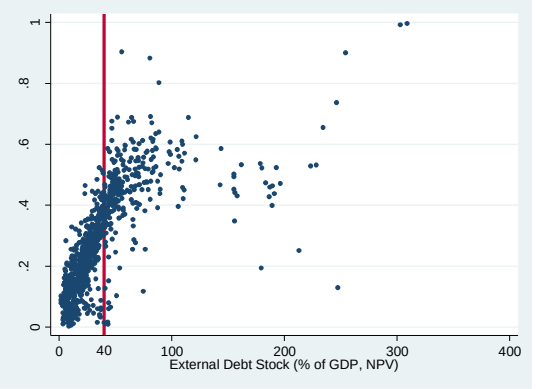


Figure 2-12: External Debt % of GDP
(Dependent Variable: *default(serial)*
Model: Probit Random-Effect)

Note: The red reference line (40%) is the World Bank's Debt Sustainability Framework's threshold for Medium policy country.

Table 2-8 explores the sensitivity of the results by using the export, instead of

GDP, as a denominator of *External Debt Stock* measures.

Table 2-8: Estimation Results for External Debt Stock (% of Export)

X_{it} \ Y_{it}	Logit				Probit	
	Fixed Effect Model		Random Effect Model		Random Effect Model	
	default	*default (serial)*	*default*	*default (serial)*	*default*	*default (serial)*
External Debt Stock (% of export)	0.018 [0.000]***	0.018 [0.000]***	0.018 [0.000]***	0.018 [0.000]***	0.011 [0.000]***	0.011 [0.000]***
External Debt Stock (quadratic) (% of export)	-3.67e-05 [0.000]***	-3.52e-05 [0.000]***	-3.70e-05 [0.000]***	-3.52e-05 [0.000]***	-2.22e-05 [0.000]***	-2.21e-05 [0.000]***
External Debt Stock (cubic) (% of export)	2.22e-08 [0.001]***	2.07e-8 [0.003]***	2.27e-08 [0.001]***	2.10e-08 [0.002]***	1.35e-08 [0.001]***	1.32e-08 [0.001]***
Debt Service Ratio (% of export)	-0.012 [0.098]*	-0.014 [0.060]*	-0.018 [0.008]***	-0.021 [0.003]***	-0.011 [0.007]***	-0.012 [0.004]***
GDP growth (annual %)	-0.018 [0.072]*	-0.031 [0.004]***	-0.022 [0.030]**	-0.035 [0.001]***	-0.013 [0.026]**	-0.021 [0.001]***
Inflation Rate, GDP deflator (annual %)	0.001 [0.093]*	0.0003 [0.245]	0.001 [0.111]	0.0002 [0.263]	0.0005 [0.097]*	0.0001 [0.224]
Short-term debt (% of total external debt)	0.015 [0.016]**	0.029 [0.000]***	0.009 [0.133]	0.018 [0.005]***	0.005 [0.150]	0.010 [0.006]***
Total Reserve (% of total external debt)	-0.0001 [0.919]	0.00003 [0.980]	-0.001 [0.451]	-0.002 [0.329]	-0.0003 [0.486]	-0.0002 [0.673]
LIC (1=Low Income Countries)	-0.544 [0.007]***	-0.669 [0.002]***	-0.290 [0.087]*	-0.383 [0.034]**	-0.167 [0.098]*	-0.213 [0.043]**
Official Creditor Commitment (million USD)	-0.0001 [0.054]*	-0.0001 [0.248]	-0.0003 [0.000]***	-0.0003 [0.001]***	-0.0001 [0.000]***	-0.0002 [0.001]***
Foreign Direct Investment (% of GDP)	-0.036 [0.010]***	-0.059 [0.000]***	-0.033 [0.011]**	-0.048 [0.002]***	-0.020 [0.010]**	-0.029 [0.001]***
Remittance (% of GDP)	-0.011 [0.422]	-0.049 [0.009]***	-0.013 [0.198]	-0.031 [0.013]**	-0.008 [0.187]	-0.018 [0.010]**
Government Expenditure (% of GDP)	-0.00002 [0.999]	-0.014 [0.330]	0.016 [0.171]	0.008 [0.537]	0.010 [0.160]	0.005 [0.519]
observation	2,288	2,127	2,482	2,473	2,482	2,473
Hausman test (H_0: Random-Effect Model is correct.)		85.47 [0.000]***	-	-	-	-
Default Probability Decreasing Zone	362 – 740%	418 – 716%	368 – 718%	399 – 718%	370 –726%	390 – 726%

Note: p-value is reported in [ ] and *, **, *** denotes significance at the 10%, 5%, and 1% level.

It shares the common results with Table 2-7, and indicates that *External Debt Stock (% of export)* remain to be statistically significant with expected signs for each in original, quadratic and cubic terms. The relationship between estimated default probability and external debt stock is plotted in Figure 2-13 through Figure 2-18, which exhibits non-monotonic relationship between debt stock and default probability, although it is not as explicit as in Figure 2-7 through Figure 2-12.

The estimated default probability decreasing zone is located at the range of 362-740% of the external debt stock % of the export. Again, this is significantly high, comparing to the threshold of the World Bank's Debt Sustainability Framework, such as 150% for the Medium strength policy country.

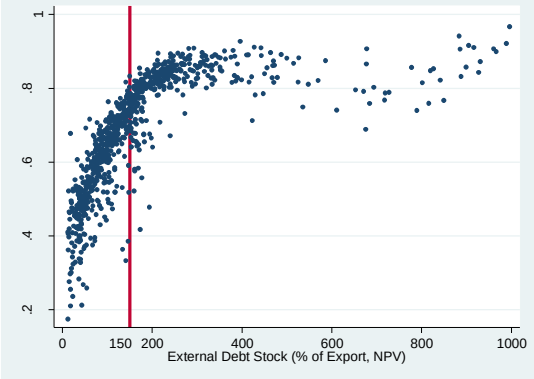


Figure 2-13: External Debt % of Export
(Dependent Variable: *default*
Model: Logit Fixed-Effect)

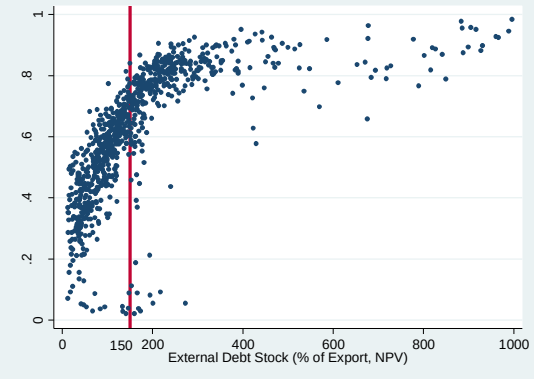


Figure 2-14: External Debt % of Export
(Dependent Variable: *default(serial)*
Model: Logit Fixed-Effect)

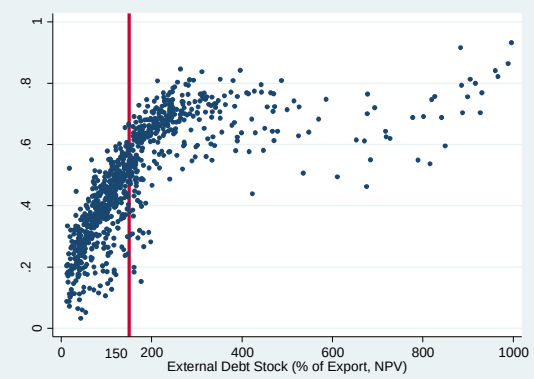


Figure 2-15: External Debt % of Export
(Dependent Variable: *default*
Model: Logit Random-Effect)

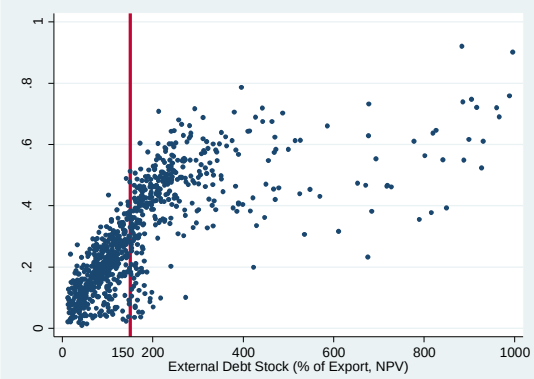


Figure 2-16: External Debt % of Export
(Dependent Variable: *default(serial)*
Model: Logit Random-Effect)

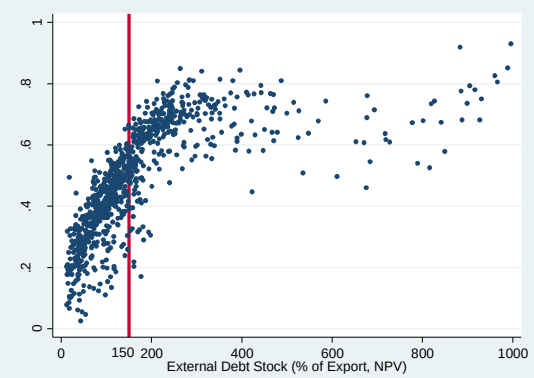


Figure 2-17: External Debt % of Export
(Dependent Variable: *default*
Model: Probit Random-Effect)

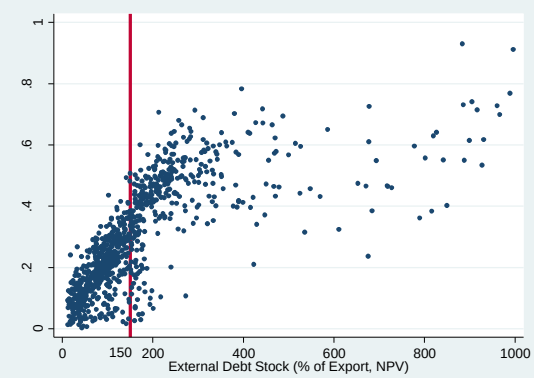


Figure 2-18: % of Export
(Dependent Variable: *default(serial)*
Model: Probit Random-Effect)

Note: The red reference line (150%) is the World Bank's Debt Sustainability Framework's threshold for Medium policy country.

Finally, in order to take a close look at which countries are actually

experiencing the decreasing default probability, the estimated results are decomposed into Asia, Sub-Sahara Africa, and others[2]. Figure 2-19 plots the estimated default probability for each region by using Logit fixed-effect model. Asia is located almost entirely left tail of the curve, which indicates that Asia is experiencing the traditional monotonic relation between the debt stock and the default probability. On the other hand, other regions especially Sub-Sahara Africa is experiencing the non-monotonic relation between the debt stock and the default probability.

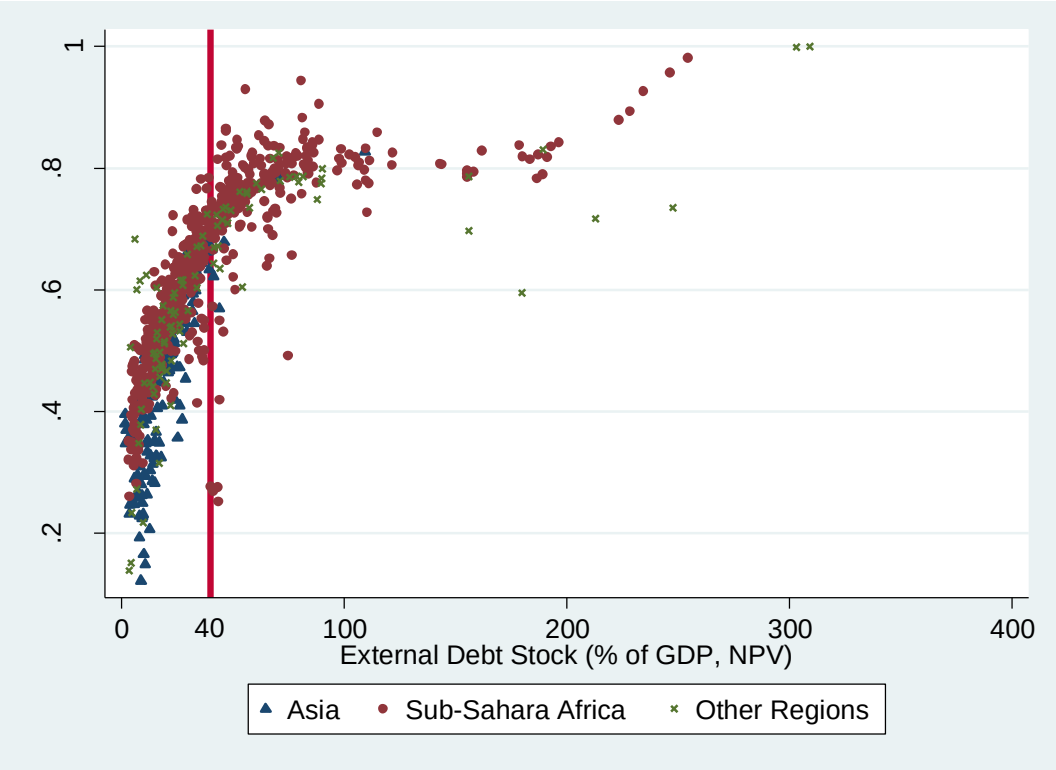


Figure 2-19: Estimated Default Probability for Asia and non-Asia

4.2. Robustness Check

Table 2-9 presents the panel unit root test, based on the Augmented Dickey-

[2] Classification is based on the World Bank's regional classification. "Asia" is defined by the countries classified as East Asia and Pacific, or South Asia by the World Bank, "Sub-Sahara Africa" is defined by the countries classified as Sub-Sahara Africa by the World Bank.

Fuller test, for the debt stock variables against the null hypothesis that all panels (i.e. country) includes unit root. For the external debt stock % of GDP, all test scores reject the null, which indicates at least one panel is stationary. For the external debt stock % of export, all test scores reject the null. Thus, the debt stock related variables are assumed to be stationary enough for panel analyses.

Table 2-9: Panel Unit Root Test

	External Debt Stock (% of GDP)	External Debt Stock (% of Export)
Inverse Chi-Squared	292.80 [0.039]**	329.61 [0.001]***
Inverse Normal	-2.38 [0.009]***	-1.67 [0.048]**
Inverse Logit	-2.96 [0.002]***	-2.92 [0.002]***
Modified Inverse Chi-Squared	1.82 [0.035]**	3.56 [0.000]***
Number of Panels	126	125
Number of Lags	1	1

Note: p-value is reported in [ ] and *, **, *** denotes significance at the 10%, 5%, and 1% level.

Table 2-10 presents the estimated results of robustness checks for the external debt stock % of GDP. Model (1) use the one-period lagged external debt stock as regressors instead of the current ones. The external debt stock related variables remains their expected sign with statistical significance. Since the external debt stock variables preserve their expected sign and statistical significance up to the three-period lag, they could be used to forecast the default for the three periods ahead. Model (2) relaxes the assumption on the no autocorrelation of the error term, and applies AR(1). Heavily Indebted Poor Countries (HIPCs)' large volume of debt and their eligibility to receive exceptionally large amount of external aid as well as debt reduction might be disturbing the empirical results. Thus, Model (3) presents the estimated results, excluding the observations of 40 HIPCs. Still, most of the external debt stock related variables remain their expected sign with statistical significance. Model (4) applies the

dynamic model by adding the one-period lagged dependent variable into regressors. Though the lagged dependent variable enters with the positive and statistically significant coefficient, all the external debt stock related variables also remains their expected sign with statistical significance.

Among the other control variables, the official creditors' commitment has negative and statistically significant coefficient in all models, as it does in the original model. GDP growth also remains to have negative and statistically significant impact in most of the models.

In summary, the external debt stock related variables maintain their statistical significance with expected sign for original, quadratic and cubic terms in most of the alternative specifications.

Table 2-10: Robustness Checks for External Debt Stock (% of GDP)

X_{it} / Y_{it}	Model (1) Logit Fixed Effect, Lagged(1) Debt Stock		Model (2) AR(1) autocorrelation, *default*		Model (3) Excluding HIPCs, *default*		Model (4) Dynamic, *default*	
	default	*default (serial)*	Logit	Probit	Logit, Fixed	Probit	Logit, Fixed	Probit
External Debt Stock (% of GDP)	0.034 [0.001]***	0.050 [0.000]***	0.048 [0.000]***	0.031 [0.000]***	0.103 [0.002]***	0.068 [0.000]***	0.053 [0.000]***	0.032 [0.000]***
External Debt Stock (quadratic) (% of GDP)	-3.00e-04 [0.016]**	-4.33e-04 [0.002]***	-3.92e-04 [0.014]**	-2.49e-04 [0.007]***	-0.001 [0.130]	-8.41e-04 [0.053]*	-4.37e-04 [0.003]***	-2.83e-04 [0.001]***
External Debt Stock (cubic) (% of GDP)	7.91e-07 [0.051]*	1.08e-06 [0.006]***	9.59e-07 [0.085]*	6.06e-07 [0.059]*	4.09e-06 [0.420]	3.30e-06 [0.260]	1.12e-06 [0.021]**	7.31e-07 [0.007]***
Debt Service Ratio (% of export)	0.005 [0.368]	0.003 [0.688]	-0.012 [0.040]**	-0.007 [0.041]**	-0.002 [0.789]	-0.004 [0.393]	-0.002 [0.737]	-0.004 [0.247]
GDP growth (annual %)	-0.032 [0.001]***	-0.047 [0.000]***	-0.023 [0.022]**	-0.014 [0.018]**	-0.009 [0.506]	-0.008 [0.327]	-0.022 [0.030]**	-0.014 [0.016]**
Inflation Rate, GDP deflator (annual %)	0.001 [0.089]*	0.0003 [0.219]	0.001 [0.044]**	0.001 [0.029]**	0.001 [0.192]	0.0004 [0.128]	0.001 [0.181]	0.0004 [0.143]
Short-term debt (% of total external debt)	0.014 [0.026]**	0.029 [0.000]***	0.003 [0.562]	0.002 [0.581]	0.008 [0.288]	0.004 [0.390]	0.016 [0.012]	0.005 [0.141]
Total Reserve (% of total external debt)	-0.001 [0.374]	-0.003 [0.215]	-0.007 [0.001]***	-0.004 [0.001]***	0.0004 [0.534]	0.0001 [0.715]	-0.0004 [0.671]	-0.001 [0.280]
LIC (1=Low Income Countries)	-0.422 [0.041]**	-0.602 [0.006]***	0.093 [0.473]	0.056 [0.482]	-0.551 [0.083]*	-0.208 [0.183]	-0.505 [0.015]**	-0.028 [0.764]
Official Creditors' Commitment (million USD)	-0.0002 [0.021]**	-0.0001 [0.136]	-0.0003 [0.000]***	-0.0002 [0.000]***	-0.0002 [0.036]**	-0.0001 [0.004]***	-0.0002 [0.023]**	-0.0001 [0.000]***
default Lagged (1)							0.444 [0.000]***	0.433 [0.000]***
Foreign Direct Investment (% of GDP)	-0.047 [0.001]***	-0.067 [0.000]***	-0.022 [0.060]*	-0.014 [0.049]**	-0.034 [0.053]*	-0.018 [0.070]*	-0.038 [0.008]***	-0.022 [0.004]***
Remittance (% of GDP)	-0.006 [0.652]	-0.048 [0.009]***	-0.011 [0.111]	-0.007 [0.083]*	-0.015 [0.363]	-0.007 [0.346]	-0.007 [0.644]	-0.005 [0.365]
Government Expenditure (% of GDP)	-0.006 [0.689]	-0.019 [0.191]	-0.001 [0.959]	0.0002 [0.970]	-0.001 [0.971]	0.005 [0.655]	-0.006 [0.660]	0.004 [0.596]
observation	2,324	2,161	2,171	2,171	1,534	1,721	2,317	2,507
Default Probability Decreasing Zone	86 – 167%	84 – 184%	93 – 180%	94 – 181%	70 – 119%	67 – 101%	94 – 166%	85 – 173%

Note: p-value is reported in [ ] and *, **, *** denotes significance at the 10%, 5%, and 1% level.

5. Policy Implications and Way Forward

Two key policy implications could be derived from the results.

Firstly, the empirical results indicate the possibility that there exist a non-monotonic relationship between the external debt stock and the default probability, which remains to be significant even after controlling the external assistance related variables. Thus, even though the international donors' involvement do reduce the default probability, the external debt stock does affect the default probability in a non-monotonic way. Hence, it indicates the possibility that some countries may be able to reduce their default probability with higher debt stock, possibly through economic growth, than the World Bank's Debt Sustainability Framework assumes. From this perspective, the current Debt Sustainability Framework is constraining the opportunities to grow for some countries. Considering that all Asian countries are located at the left side of the hump of the curve, it might be Asian countries that the Debt Sustainability Framework is imposing sub-optimal constraints. If this is the case, the Debt Sustainability Framework is the system to preserve or improve the debt sustainability of Sub-Sahara African countries (and others) by sacrificing the economic growth opportunities of Asian countries. It is understandable and reasonable for the World Bank, as a leading global institution, to establish the conservative standard, since the World Bank is in the responsible position to help the least developed countries as well as to sustain the stability of the global financial structure. However, if this sublime goal is being realized by shifting the forgone opportunities costs to grow onto some specific countries or regions, there seems a room to reconsider the system as a whole.

On the other hand, it is also true that the non-monotonicity of default probability should be interpreted with careful considerations. Considering that large part of the predicted default events occur before the estimated default probability

decreasing zone, and that the level which the estimated default probability decrease is far beyond the threshold of the Debt Sustainability Framework, it is too reckless to conclude that the higher debt would be acceptable. In addition, what causes the geographic difference is not clear. If the non-monotonicity is indeed caused by the productive public capital, as developed in Section 3, the geographic difference seems counter-intuitive, since typically the productivity of the public capital is supposed to higher in Asia than in Sub-Sahara Africa.

Second policy implication is that the empirical results indicates a moral hazard problem of the heavily indebted countries, since decreasing default probability is explained by the external assistance to some extent. Thus, the more indebted the countries are, the more aid they expect to receive from the international donor community as a bailout. Through the series of debt relief, HIPC Initiative in 1996, Enhanced HIPC Initiative in 1999, and most recently the Multilateral Debt Relief Initiative (MDRI) in 2005, the international donor community has been trying to apply the strict eligibility of debt relief and providing close monitoring of the economic management of the post-debt relief countries. The further analysis needs to be done to capture how these donors' efforts affect the borrowers' behavior.

Since the derived empirical results heavily depend on (i) the definition of the sovereign default, and (ii) the functional forms of the models, there are three possible directions to move the research further.

Firstly, it is necessary to construct the data set of alternative sovereign default. Alternative sovereign default data needs to dig into the case studies of each default episodes of the low income countries, with attention to separating the overlapping default episodes. In addition, for the HIPCs, it is necessary to take into account the timing that they reach the Decision Point, the Completion Point, and the MDRI's debt relief. Besides, further analysis on the causality of the geographic difference is

necessary, in order to derive more sensible policy implications. Empirically, additional explanatory variables such as the quality of the public investment as well as the institutions and policy are necessary to control the potential geographic difference.

Secondly, non-parametric approach should be explored to complement the current models. Some of the recent works use the Classification and Regression Tree method, which is a sort of data mining technique to pick up the most predictive variable through splitting entire samples into sub-trees. It does not provide the evidence on non-monotonicity, but it could be used to show how strong explanatory power the stock-related variables may have for predicting the default events.

APPENDIX

APPENDIX 2-1: DATA DESCRIPTION AND SOURCES

OECD, DAC (Development Assistance Committee) database on Aid, 2012.

World Bank, 2012. *Global Development Finance (GDF)*, Washington, D.C.

World Bank, 2012. *World Development Indicators (WDI)*, Washington, D.C.

Variable Name	Variable Description	Data Source
default	Dummy variable which takes 1 if the arrears are increasing from the previous year	Arrears data from GDF
default (serial)	Dummy variable which takes 1 if the arrears are increasing more than two consecutive years.	Arrears data from GDF
External Debt Stock (% of GDP)	Net Present Value of External Debt Stock (% of GDP)	GDF, Transformed into Net Present Value by author's calculation
External Debt Stock (% of export)	Net Present Value of External Debt Stock (%of exports of goods, services and income)	GDF, Transformed into Net Present Value by author's calculation
Debt Service Ratio (% of export)	Debt Service (% of exports of goods, services and income)	GDF
GDP growth (annual %)	GDP growth rate (annual %)	WDI
Inflation Rate	Inflation, GDP deflator (annual %)	WDI
Short-term debt (% of total external debt)	Short-term debt (% of total external debt)	WDI
Total Reserve (% of total external debt)	Total Reserve (% of total external debt)	WDI
LIC	Dummy variable which takes 1 if the country is classified as the low income country based on the World Bank's definition	WDI
Official Creditors' Commitment (million USD)	Amount of long-term from official creditors including loans from international organizations (multilateral loans) and loans from governments (bilateral loans, official export credits).	GDF

APPENDIX 2-2: SUMMARY STATISTICS

Variable	Mean	Std. Dev.	Min	Max
	Entire Samples			
default	0.45	0.50	0	1
default (serial)	0.29	0.45	0	1
External Debt Stock (% of GDP)	29.02	35.51	0	627.44
External Debt Stock (% of Export)	128.72	208.44	0	3,737.67
Debt Service Ratio	13.51	11.94	0	150.84
GDP growth	3.77	6.55	-51.03	106.28
Inflation Rate	60.33	612.24	-29.17	26,762.02
Short-term debt	11.97	12.55	0	88.93
Total Reserve	58.28	249.03	-0.17	6,517.15
LIC	0.38	0.48	0	1
Official Creditors' Commitment	467.61	1,075.76	0	23,206.90
Foreign Direct Investment	3.03	5.73	-82.89	90.74
Remittances	4.82	8.94	0.00	96.94
Government Expenditure	14.99	6.49	1.38	69.54
	Low Income Countries			
default	0.53	0.50	0	1
default (serial)	0.36	0.48	0	1
External Debt Stock (% of GDP)	39.04	42.05	0	478.66
External Debt Stock (% of Export)	212.44	303.28	0	3,737.67
Debt Service Ratio	14.11	12.99	0	150.84
GDP growth	3.64	6.91	-50.25	106.28
Inflation Rate	85.89	861.68	-27.05	26,762.02
Short-term debt	7.90	9.00	0	83.15
Total Reserve	34.51	126.46	-0.17	3,366.78
LIC	-	-	-	-
Official Creditors' Commitment	387.17	903.78	0	9,295.49
Foreign Direct Investment	2.72	6.50	-28.62	90.74
Remittances	4.60	10.11	0.001	96.94
Government Expenditure	13.78	6.97	2.05	69.54

APPENDIX 2-3: SAMPLE COUNTRIES

	Country	*World Bank's Regional Classification*	*World Bank's Classification of Income Group*	*Other*
1	Afghanistan	South Asia	Low income	HIPC
2	Albania	Europe & Central Asia	Lower middle income	
3	Algeria	Middle East & North Africa	Upper middle income	
4	Angola	Sub-Saharan Africa	Lower middle income	
5	Argentina	Latin America & Caribbean	Upper middle income	
6	Armenia	Europe & Central Asia	Lower middle income	
7	Azerbaijan	Europe & Central Asia	Lower middle income	
8	Bangladesh	South Asia	Low income	
9	Belarus	Europe & Central Asia	Upper middle income	
10	Belize	Latin America & Caribbean	Lower middle income	
11	Benin	Sub-Saharan Africa	Low income	HIPC
12	Bhutan	South Asia	Lower middle income	
13	Bolivia	Latin America & Caribbean	Lower middle income	HIPC
14	Bosnia and Herzegovina	Europe & Central Asia	Upper middle income	
15	Botswana	Sub-Saharan Africa	Upper middle income	
16	Brazil	Latin America & Caribbean	Upper middle income	
17	Bulgaria	Europe & Central Asia	Upper middle income	
18	Burkina Faso	Sub-Saharan Africa	Low income	HIPC
19	Burundi	Sub-Saharan Africa	Low income	HIPC
20	Cambodia	East Asia & Pacific	Low income	
21	Cameroon	Sub-Saharan Africa	Lower middle income	HIPC
22	Cape Verde	Sub-Saharan Africa	Lower middle income	
23	Central African Republic	Sub-Saharan Africa	Low income	HIPC
24	Chad	Sub-Saharan Africa	Low income	HIPC
25	Chile	Latin America & Caribbean	Upper middle income	
26	China	East Asia & Pacific	Lower middle income	
27	Colombia	Latin America & Caribbean	Upper middle income	
28	Comoros	Sub-Saharan Africa	Low income	HIPC
29	Congo, Dem. Rep.	Sub-Saharan Africa	Low income	HIPC
30	Congo, Rep.	Sub-Saharan Africa	Lower middle income	HIPC
31	Costa Rica	Latin America & Caribbean	Upper middle income	
32	Côte d'Ivoire	Sub-Saharan Africa	Lower middle income	HIPC
33	Croatia	..	High income: non-OECD	
34	Djibouti	Middle East & North Africa	Lower middle income	
35	Dominica	Latin America & Caribbean	Upper middle income	
36	Dominican Republic	Latin America & Caribbean	Upper middle income	
37	Ecuador	Latin America & Caribbean	Lower middle income	
38	Egypt, Arab Rep.	Middle East & North Africa	Lower middle income	
39	El Salvador	Latin America & Caribbean	Lower middle income	
40	Eritrea	Sub-Saharan Africa	Low income	HIPC
41	Ethiopia	Sub-Saharan Africa	Low income	HIPC
42	Fiji	East Asia & Pacific	Upper middle income	
43	Gabon	Sub-Saharan Africa	Upper middle income	
44	Gambia, The	Sub-Saharan Africa	Low income	HIPC
45	Georgia	Europe & Central Asia	Lower middle income	
46	Ghana	Sub-Saharan Africa	Low income	HIPC
47	Grenada	Latin America & Caribbean	Upper middle income	
48	Guatemala	Latin America & Caribbean	Lower middle income	
49	Guinea	Sub-Saharan Africa	Low income	HIPC
50	Guinea-Bissau	Sub-Saharan Africa	Low income	HIPC
51	Guyana	Latin America & Caribbean	Lower middle income	HIPC
52	Haiti	Latin America & Caribbean	Low income	HIPC
53	Honduras	Latin America & Caribbean	Lower middle income	HIPC
54	India	South Asia	Lower middle income	
55	Indonesia	East Asia & Pacific	Lower middle income	
56	Iran, Islamic Rep.	Middle East & North Africa	Lower middle income	
57	Jamaica	Latin America & Caribbean	Upper middle income	
58	Jordan	Middle East & North Africa	Lower middle income	
59	Kazakhstan	Europe & Central Asia	Upper middle income	
60	Kenya	Sub-Saharan Africa	Low income	
61	Kyrgyz Republic	Europe & Central Asia	Low income	HIPC
62	Lao PDR	East Asia & Pacific	Low income	
63	Latvia	Europe & Central Asia	Upper middle income	
64	Lebanon	Middle East & North Africa	Upper middle income	
65	Lesotho	Sub-Saharan Africa	Lower middle income	
66	Liberia	Sub-Saharan Africa	Low income	HIPC
67	Macedonia, FYR	Europe & Central Asia	Upper middle income	
68	Madagascar	Sub-Saharan Africa	Low income	HIPC
69	Malawi	Sub-Saharan Africa	Low income	HIPC
70	Malaysia	East Asia & Pacific	Upper middle income	
71	Maldives	South Asia	Lower middle income	
72	Mali	Sub-Saharan Africa	Low income	HIPC
73	Mauritania	Sub-Saharan Africa	Low income	HIPC
74	Mauritius	Sub-Saharan Africa	Upper middle income	
75	Mexico	Latin America & Caribbean	Upper middle income	
76	Moldova	Europe & Central Asia	Lower middle income	
77	Mongolia	East Asia & Pacific	Lower middle income	
78	Montenegro	Europe & Central Asia	Upper middle income	
79	Morocco	Middle East & North Africa	Lower middle income	
80	Mozambique	Sub-Saharan Africa	Low income	HIPC
81	Myanmar	East Asia & Pacific	Low income	
82	Nepal	South Asia	Low income	
83	Nicaragua	Latin America & Caribbean	Lower middle income	HIPC
84	Niger	Sub-Saharan Africa	Low income	HIPC
85	Nigeria	Sub-Saharan Africa	Lower middle income	
86	Pakistan	South Asia	Lower middle income	
87	Panama	Latin America & Caribbean	Upper middle income	
88	Papua New Guinea	East Asia & Pacific	Lower middle income	
89	Paraguay	Latin America & Caribbean	Lower middle income	
90	Peru	Latin America & Caribbean	Upper middle income	
91	Philippines	East Asia & Pacific	Lower middle income	
92	Poland	Europe & Central Asia	Upper middle income	
93	Romania	Europe & Central Asia	Upper middle income	
94	Russian Federation	Europe & Central Asia	Upper middle income	
95	Rwanda	Sub-Saharan Africa	Low income	HIPC
96	Samoa	East Asia & Pacific	Lower middle income	
97	São Tomé and Principe	Sub-Saharan Africa	Lower middle income	HIPC
98	Senegal	Sub-Saharan Africa	Low income	HIPC
99	Serbia	Europe & Central Asia	Upper middle income	
100	Seychelles	Sub-Saharan Africa	Upper middle income	
101	Sierra Leone	Sub-Saharan Africa	Low income	HIPC
102	Solomon Islands	East Asia & Pacific	Lower middle income	
103	Somalia	Sub-Saharan Africa	Low income	HIPC
104	South Africa	Sub-Saharan Africa	Upper middle income	
105	Sri Lanka	South Asia	Lower middle income	
106	St. Kitts and Nevis	Latin America & Caribbean	Upper middle income	
107	St. Lucia	Latin America & Caribbean	Upper middle income	
108	St. Vincent and the Grenadines	Latin America & Caribbean	Upper middle income	
109	Sudan	Sub-Saharan Africa	Lower middle income	HIPC
110	Swaziland	Sub-Saharan Africa	Lower middle income	
111	Syrian Arab Republic	Middle East & North Africa	Lower middle income	
112	Tajikistan	Europe & Central Asia	Low income	
113	Tanzania	Sub-Saharan Africa	Low income	HIPC
114	Thailand	East Asia & Pacific	Lower middle income	
115	Togo	Sub-Saharan Africa	Low income	HIPC
116	Tonga	East Asia & Pacific	Lower middle income	
117	Tunisia	Middle East & North Africa	Lower middle income	
118	Turkey	Europe & Central Asia	Upper middle income	
119	Turkmenistan	Europe & Central Asia	Lower middle income	
120	Uganda	Sub-Saharan Africa	Low income	HIPC
121	Ukraine	Europe & Central Asia	Lower middle income	
122	Uruguay	Latin America & Caribbean	Upper middle income	
123	Uzbekistan	Europe & Central Asia	Low income	
124	Vanuatu	East Asia & Pacific	Lower middle income	
125	Venezuela, RB	Latin America & Caribbean	Upper middle income	
126	Vietnam	East Asia & Pacific	Low income	
127	Yemen, Rep.	Middle East & North Africa	Low income	
128	Zambia	Sub-Saharan Africa	Low income	HIPC
129	Zimbabwe	Sub-Saharan Africa	Low income	

Note: Income classifications are in effect until 1 July 2010

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CHAPTER 3

DYNAMIC GENERAL EQUILIBRIUM ANALYSIS ON THE WORLD BANK'S TRAFFIC LIGHT SYSTEM

1. Introduction

In 2004, the World Bank (IDA) introduced the Debt Sustainability Framework (DSF) for low-income countries, so called "traffic light system", based on which IDA determines the grant/loan ratio to be allocated to each recipient country for each fiscal year. For instance, the country receives 100% grant aid, if the country is classified as "red light" (i.e. its debt level is unsustainable), under the traffic light system. The World Bank and IMF have been requesting the other institutions (aid donors, export credit agencies and private lenders) to comply with this system, in order to maintain developing countries' debt sustainability and to avoid another debt crises.

As suggested by the empirical results of the chapter 2, the current thresholds for external borrowing under the traffic light system could be sub-optimal for some countries with high potential to achieve economic growth by utilizing debt-financed investment. If this is the case, the traffic light system might be unduly constraining the ability of developing countries to finance their development goals. This leads to the motivation of this paper.

The compositional change of the traffic lights is summarized in Figure 3-1. In FY2005, "red" light countries were more than half of total IDA-only countries, but have been declining year by year. In FY2013, "red" light countries are one third of IDA-only countries. The frequency of the traffic light's change is summarized in Table 3-1. Although the traffic light becomes relatively stable, especially after introducing three year moving average Country Policy and Institutional Assessment

(CPIA) index in FY2008, around ten countries per year face up with the traffic light's change. Figure 3-2 shows some of the extreme cases; Nepal and Djibouti. Considering that neither of them are Heavily Indebted Poor Countries (HIPCs) and thus are not affected by debt reduction, its traffic light's volatility stems, not from the debt stock change but from the institutional problems with the traffic light system.

Table 3-1: Change of Traffic Lights

	FY2006	FY2007	FY2008	FY2009	FY2010	FY2011	FY2012	FY2013
Countries with Changed Traffic Light	16	19	11	7	5	7	9	8
IDA-only Countries	66	66	65	64	64	63	64	64

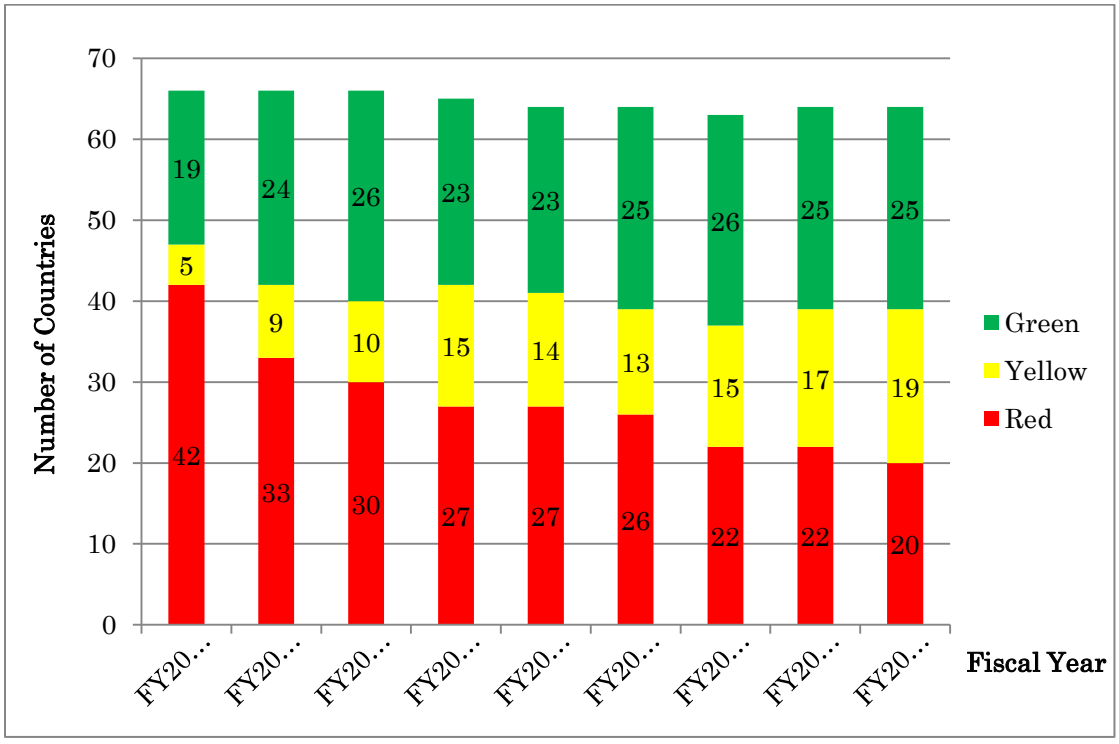

Figure 3-1: Compositional Change of Traffic Lights

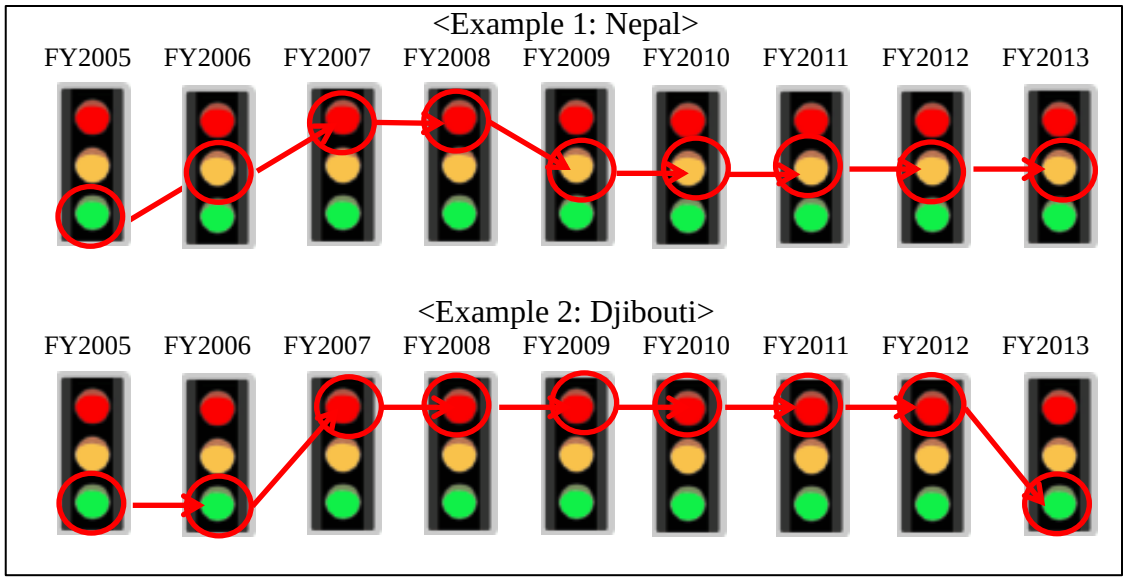

Figure 3-2: Example of Countries with Frequent Traffic Light Changes

This paper's objective is to analyze how the traffic light system affects the dynamic general equilibrium of the recipient countries. My hypothesis is that the traffic light system leads to (i) binding budget constraint, and (ii) volatility of aid inflow (in terms of both the volume as well as the grant ratio), both of which eventually reduces the recipient countries' long-term growth.

This paper is organized as follows. The next section reviews the related literatures and provides a general direction of detailed analysis. The third section describes the theoretical model. The fourth section presents the numerical analysis. The last section provides some policy implications and way forward.

2. Literature Review

Public investment, especially infrastructure investment, has been widely recognized as the important factor of economic growth for the developing countries, and thus their governments have been putting enormous efforts on infrastructure development. As described in the World Bank (1994), infrastructure in low- and

middle-income countries typically represents about 20% of total investment and 40-60% of public investment, and nearly half of all public investment in infrastructure is underwritten by the donor financing. Thus, (i) the economic growth, (ii) the public investment, and (iii) the external financing by the foreign donors are closely interrelated in the developing countries.

There are two relevant strands of the past works on the above-mentioned interrelation. First strand focus on the link between the economic growth and the public investment. The theoretical foundations of the role of public capital on private capital accumulation and economic growth date back to the seminal paper of Arrow and Kurz (1970). The subsequent works, such as Aschauer (1989a, 1989b), seek to demonstrate that publicly provided capital should enter as a complementary input to private production. Barro (1990) develops a model of endogenous growth in which the government uses tax revenue to finance government expenditure and this expenditure (i.e. public investment) enters into the production function as a productive input. While his model essentially reduces to a version of the "AK model" in which there are no transitional dynamics, Futagami et al. (1993) develops an endogenous growth model with productive public capital along with private capital in a spirit similar to that of Barro (1990) but with a transitional dynamics. Most of these previous literatures use the domestic tax revenue, instead of the external financing, as a fiscal source of public investment.

Second strand examines the link between the economic growth and the external financing, especially the foreign aid. Large volume of past works have been done, especially on the presumably positive impact of the official aid on the economic growth of the developing countries. One of the recent trend in this field focus on the aid volatility and predictability. Bulir and Hamann (2003) examines empirical evidence on the volatility and uncertainty of aid flows. One of their findings is that aid

is found to be more volatile than fiscal revenues and that uncertainty about aid disbursement is large. Arellano, Bulir, Lane and Lipschitz (2008) examine the effects of aid and its volatility on consumption, investment, and the structure of production in the context of an intertemporal two-sector general equilibrium model, calibrated using data for aid-dependent countries in Africa. One of their findings is that aid volatility results in substantial welfare losses, providing a motivation for a recent discussion of aid architecture stressing the need for greater predictability. Peiris and Saxegaard (2007) evaluate monetary policy-tradeoffs in low-income countries using a dynamic stochastic general equilibrium (DSGE) model estimated on data for Mozambique taking into account the sources of major exogenous shocks, and level of financial development. One of their findings is that the aid leads to an increase in the demand for non-tradable goods as well as imports. As a result of the former there is an increase in labor as well as GDP, while the increased demand for imports leads to a deterioration in the trade balance.

Until recently there are limited attempts to incorporate all three factors, (i) the economic growth, (ii) the public investment, and (iii) the external financing, into one model with a view to apply to the low income countries. The Gleneagles G8 Summit in 2005 accelerate the necessity of the extensive research in this area, since one of its outcomes is scaling-up of aid, with further debt cancellation, to Africa. Since then, work is underway to construct formal general equilibrium model for the low income countries to assess the complex interlinkages among the macroeconomic impact of large aid inflow, public investment, and growth. Chatterjee, Sakoulis and Turnovsky (2003) analyze the effects of grant aid, which is tied to public infrastructure (productive public expenditure), on economic growth and welfare. One of their findings is that those kind of aid generates dynamic adjustments, as public capital accumulated in the recipient country. Agenor and Yilmaz (2008) develops the model

based on productive public capital, and examine how the permanent and temporary increase of aid affects the growth and the welfare of the typical low income countries. Berg et al. (2010) develops a New-Keynesian model for Uganda and examine how different fiscal and monetary policies can affect the "spending" and "absorption" of aid. Buffie et al. (2012) develops a model to study the macroeconomics effects of public investment surges in low-income countries, making explicit: (i) the investment-growth linkages, (ii) public external and domestic debt accumulation, (iii) the fiscal policy reactions necessary to ensure debt-sustainability, and (iv) the macroeconomic adjustment required to ensure internal and external balance.

This paper shares the similar motivation with the above-mentioned continuing works to analyze the growth – public investment nexus underpinning the World Bank's Debt Sustainability Framework.

3. Theoretical Model

The theoretical model is constructed by incorporating "productive public expenditure", which is financed by domestic financing and external grant/loan aid, into the dynamic general equilibrium. I consider a small open economy populated by an infinitely lived representative agent who produces and consumes a single traded commodity.

3.1. Private Sector

Output of the commodity is produced by the constant returns to scale production function;

$$Y = \alpha \left(\frac{K_G}{K_P}\right)^{\eta} K_P = \alpha K_G^{\eta} K_P^{1-\eta} \ , \quad \alpha > 0, \qquad 0 < \eta < 1 \tag{1}$$

where

Y: output

K_G: public capital stock

K_P: private capital stock

Thus, equation (1) assumes that the public capital stock enhances the productivity of private capital stock.

The agent consumes this good, yielding utility over an infinite horizon represented by the isoelastic utility function;

$$U \equiv \int_0^{\infty} \frac{1}{\gamma} C^{\gamma} e^{-\beta t} dt \quad , \quad -\infty < \gamma < 1 \tag{2}$$

where

C: consumption

The agent accumulates its private capital, with expenditure on a given change in the private capital stock, involving adjustment (installation) costs specified by the quadratic function;

$$\Psi_P(I_P, K_P) = I_P + h_P \frac{I_P^2}{2K_P} = I_P \left(1 + \frac{h_P}{2} \frac{I_P}{K_P}\right) \tag{3}$$

where

I_P: change in private capital stock (new private capital investment)

The net rate of private capital accumulation is defined as;

$$\dot{K_P} = I_P - \delta_P K_P \tag{4}$$

where

δ_P: depreciation rate of private capital

The agent can borrow from international credit market, and its cost of borrowing depends on its creditworthiness of the economy. The international credit market is assumed to assess the economy's ability to repay and default risk by the country's debt-capital (equity) ratio. Therefore, the interest rate, which the county is

charged, increases with the debt-capital ratio, which leads to upward-sloping supply schedule of debt;

$$r\left(\frac{D}{K}\right) = r^* + \omega\left(\frac{D}{K}\right), \quad \omega' > 0 \tag{5}$$

where

D: national debt stock $(= D_P + D_G)$

r^*: exogenously given world interest rate

K: national capital stock $(= K_P + K_G)$

$\omega\left(\frac{D}{K}\right)$: country-specific borrowing premium that increases with national debt-capital ratio

It should be noted that the agent takes the interest rate, equation (5), as given, since it is a function of the country's aggregate (i.e. national) debt-capital ratio, which the agent is assumed to be unable to influence.

The agent's decision problem is to choose his rates of consumption, C, private investment, I_P, and accumulation of private debt, D_P, as well as private capital, K_P, to maximize the intertemporal utility, equation (2), subject to the flow budget constraint;

$$\dot{D_P} = C + r\left(\frac{D}{K}\right) D_P + \Psi_P(I_P, K_P) - (1-\tau)Y + T \tag{6}$$

where

D_P: private debt stock

τ: income tax

T: lump-sum tax

The discounted Hamiltonian for this dynamic optimization is;

$$\begin{aligned} H \equiv\ & e^{-\beta t}\frac{1}{\gamma}C^{\gamma} \\ & -\mu_1\left[C + r\left(\frac{D}{K}\right) D_P + \Psi_P(I_P, K_P) - (1-\tau)Y + T\right] + \mu_2[I_P - \delta_P K_P] \end{aligned} \tag{7}$$

$$= e^{-\beta t}\left[\frac{1}{\gamma}C^{\gamma} - \mu_1 e^{\beta t}\left[C + r\left(\frac{D}{K}\right)D_P + \Psi_P(I_P, K_P) - (1-\tau)Y + T\right]\right.$$

$$\left. + \mu_2 e^{\beta t}[I_P - \delta_P K_P]\right]$$

where

μ_1: discounted shadow value of wealth (i.e. negative of debt) in the form of internationally traded bonds

μ_2: discounted shadow value of agent's private capital stock

Define the current-value multiplier as;

$v = \mu_1 e^{\beta t}$: current shadow value of wealth in the form of internationally traded bonds

$q' = \mu_2 e^{\beta t}$: current shadow value of agent's private capital stock

Define also the current value Hamiltonian, H^*;

$$H^* = e^{\beta t}H = \frac{1}{\gamma}C^{\gamma} - v\left[C + r\left(\frac{D}{K}\right)D_P + \Psi_P(I_P, K_P) - (1-\tau)Y + T\right] + q'[I_P - \delta_P K_P] \quad (8)$$

Pontryagin's principle indicates that maximization of the Hamiltonian by choice of C and I_P sequence leads to optimality conditions;

$$\frac{\partial H}{\partial C} = 0 \;\rightarrow\; C^{\gamma-1} = v \quad (9)$$

$$\frac{\partial H}{\partial I_P} = 0 \;\rightarrow\; 1 + h_P\frac{I_P}{K_P} = q \quad (10)$$

where

$q = \frac{q'}{v}$: market price of private capital in terms of price of foreign bonds

Equation (9) equates the marginal utility of consumption to the shadow value of wealth, while equation (10) equates the marginal cost of an additional unit of investment, which is inclusive of the marginal installation cost, $h_P\frac{I_P}{K_P}$, to the market

value of capital. Equation (10), combined with equation (4), leads to the following expression for the rate of private capital accumulation;

$$\frac{\dot{K_P}}{K_P} \equiv \phi_P = \frac{q-1}{h_P} - \delta_P \tag{11}$$

Applying the standard optimality conditions with respect to D_P and K_P implies the usual arbitrage relationships, equating the rates of return on consumption and investment in private capital to the costs of borrowing abroad;

$$\begin{aligned}
\dot{v} &= \beta v + \dot{\mu_1} e^{\beta t} \\
&= \beta v - e^{\beta t} \frac{\partial H}{\partial D_P} \\
&= \beta v - e^{\beta t} \mu_1 r \left(\frac{D}{K}\right) \\
&= \beta v - e^{\beta t} v e^{-\beta t} r \left(\frac{D}{K}\right) \\
&= \beta v - v r \left(\frac{D}{K}\right) \\
&\rightarrow \ \beta - \frac{\dot{v}}{v} = r \left(\frac{D}{K}\right)
\end{aligned} \tag{12}$$

$$\begin{aligned}
\dot{q'} &= \beta q' + \dot{\mu_2} e^{\beta t} \\
&= \beta q' - e^{\beta t} \frac{\partial H}{\partial K_P} \\
&= \beta q' - e^{\beta t} \left[-\mu_1 \frac{\partial \Psi}{\partial K_P} + \mu_1 (1-\tau) \frac{\partial Y}{\partial K_P} - \mu_2 \delta_P \right] \\
&= \beta q' - e^{\beta t} \left[-\left(v e^{-\beta t}\right) \left(-\frac{h_P I_P^2}{2K_P^2} \right) + \left(v e^{-\beta t}\right)(1-\tau)(1-\eta) \alpha K_G^{\eta} K_P^{-\eta} \right. \\
&\qquad \left. - (q' e^{-\beta t}) \delta_P \right] \\
&= \beta q' - v \left(\frac{h_P I_P^2}{2K_P^2} \right) - v(1-\tau)(1-\eta) \alpha K_G^{\eta} K_P^{-\eta} + q' \delta_P
\end{aligned}$$

$$\frac{\dot{q'}}{q'} = \beta - \frac{v}{q'}\left(\frac{h_P I_P^2}{2K_P^2}\right) - \frac{v}{q'}(1-\tau)(1-\eta)\alpha K_G^{\eta} K_P^{-\eta} + \delta_P$$

$$\frac{\dot{q'}}{q'} = \beta - \frac{v}{q'}\left(\frac{h_P I_P^2}{2K_P^2}\right) - \frac{v}{q'}(1-\tau)(1-\eta)\alpha K_G^{\eta} K_P^{-\eta} + \delta_P$$

$$\frac{\frac{\partial qv}{\partial t}}{qv} = \beta - \frac{1}{q}\left(\frac{h_P(\dot{K_P} + \delta_P K_P)^2}{2K_P^2}\right) - \frac{1}{q}(1-\tau)(1-\eta)\alpha K_G^{\eta} K_P^{-\eta} + \delta_P$$

$$\frac{\dot{q}v + q\dot{v}}{qv} = \beta - \frac{1}{q}\left(\frac{h_P\left(\frac{q-1}{h_P}K_P - \delta_P K_P + \delta_P K_P\right)^2}{2K_P^2}\right) - \frac{1}{q}(1-\tau)(1-\eta)\alpha K_G^{\eta} K_P^{-\eta} + \delta_P$$

$$\frac{\dot{q}}{q} + \frac{\dot{v}}{v} = \beta - \frac{1}{q}\left(\frac{h_P(\frac{q-1}{h_P}K_P)^2}{2K_P^2}\right) - \frac{1}{q}(1-\tau)(1-\eta)\alpha K_G^{\eta} K_P^{-\eta} + \delta_P$$

$$\frac{1}{q}(1-\tau)(1-\eta)\alpha K_G^{\eta} K_P^{-\eta} + \frac{\dot{q}}{q} + \frac{1}{q}\left(\frac{(q-1)^2}{2h_P}\right) - \delta_P = \beta - \frac{\dot{v}}{v}$$

$$\rightarrow \frac{(1-\tau)(1-\eta)\alpha K_G^{\eta} K_P^{-\eta}}{q} + \frac{\dot{q}}{q} + \frac{(q-1)^2}{2h_p q} - \delta_P = r\left(\frac{D}{K}\right) \quad (13)$$

Finally, in order to ensure that the agent's intertemporal budget constraint is met, the following transversality conditions must hold;

$$\lim_{t\to\infty} D_P v e^{-\beta t} = 0, \quad \lim_{t\to\infty} K_P q' e^{-\beta t} = 0 \quad (14)$$

3.2. Public Sector

In addition to the private capital, the country accumulates the public capital as;

$$I_G = \bar{I_G} + \lambda_{GA}[\ln(1+\theta)\,A] + \lambda_{LA}[(1-\theta)A] \quad (15)$$

where

I_G: change in public capital stock (new public capital investment)

$\bar{I_G}$: new public capital investment financed domestically

A: total external aid

θ: grant ratio of external aid

λ_{GA}, λ_{LA}: degree to which each aid is tied to public capital investment ($0 \leq \lambda_{GA} \leq \lambda_{LA} \leq 1$)

Thus, equation (15) states that the public capital investment is financed by two channels: domestic finance and external aid. The case $\lambda = 1$ implies that external aid is completely tied to pubic capital investment, representing a "productive" aid, while the case $\lambda = 0$ implies that external aid is not invested in public capital and hence represent a "pure" aid. It is to be noted that the loan aid is assumed to be more tied to public capital investment than the grant aid (i.e. $\lambda_{GA} \leq \lambda_{LA}$).

In addition, the case $\theta = 1$ implies that external aid is completely grant, while the case $\theta = 0$ implies that external aid takes the form of loan. Thus, the case $\theta = 1$ is a proxy of "red light", and the case $\theta = 0$ is a proxy of "green light" under the World Bank's traffic light system. It is to be noted that the grant aid is not fully provided but that it is discounted as $\ln(1 + \theta)$. This reflects two actual phenomena. Firstly, the World Bank's traffic light system indeed discounts each country's grant allocation by 80%, in order to avoid moral hazard among the recipient countries. This discount is supposed to function as an incentive for a "red light" country to improve their debt sustainability to become a "green light" country which can receive the allocated loan aid without any discount. Secondly, other aid donors than the World Bank, especially bilateral aid donors, cannot provide as much grant aid as the World Bank can, since they do not have abundant grant resources. Therefore, as the recipient country becomes eligible for 100% grant aid, other donors face more difficulties to provide full-fledged grant. Thus, even if the recipient country is "red light" (i.e. $\theta = 1$), its actual grant amount to be received is discounted to 69% of what it should

receive if there is no restriction on grant amount.

The net rate of accumulation of public capital is defined as;

$$\dot{K}_G = I_G - \delta_G K_G \tag{16}$$

where

δ_G: depreciation rate of public capital

Analogous to the private capital, the gross accumulation of public capital is also subject to convex adjustment (installation) costs;

$$\Psi_G(I_G, K_G) = I_G + h_G \frac{I_G^2}{2K_G} = I_G\left(1 + \frac{h_G}{2}\frac{I_G}{K_G}\right) \tag{17}$$

To sustain an equilibrium of on-going growth, both new public capital investment financed domestically, $\bar{I_G}$, and the external aid flow, A, must be tied to the scale of the economy;

$$\bar{I_G} = \sigma_G Y, \quad A = \sigma_A Y, \quad \sigma_G > 0, \quad \sigma_A > 0, \quad 0 < \sigma_G + \sigma_A < 1$$

Therefore, equation (15) can be re-written as;

$$\begin{aligned} \dot{K}_G &= \sigma_G Y + \lambda_{GA}[\ln(1+\theta)\,\sigma_A Y] + \lambda_{LA}[(1-\theta)\sigma_A Y] - \delta_G K_G \\ &= [\sigma_G + \lambda_{GA}\ln(1+\theta)\,\sigma_A + \lambda_{LA}(1-\theta)\sigma_A]Y - \delta_G K_G \end{aligned} \tag{18}$$

Then, dividing equation (18) by K_G, the growth rate of public capital is given by;

$$\frac{\dot{K}_G}{K_G} \equiv \phi_G = [\sigma_G + \lambda_{GA}\ln(1+\theta)\,\sigma_A + \lambda_{LA}(1-\theta)\sigma_A]\frac{Y}{K_G} - \delta_G \tag{19}$$

The government faces the following flow budget constraint;

$$\begin{aligned} \dot{D_G} &= \Psi_G(I_G, K_G) + r\left(\frac{D}{K}\right)D_G - \tau Y - T - [\ln(1+\theta)\,A] + [(1-\theta)A] \\ &= \Psi_G(I_G, K_G) + r\left(\frac{D}{K}\right)D_G - \tau Y - T - A(\ln(1+\theta) - (1-\theta)) \end{aligned} \tag{20}$$

where

D_G: public debt stock

Equation (20) states that the excess of domestic government expenditure on

public capital and interest payments on debt over tax is financed by the tax revenue, the external aid, and accumulating public debt. If the external aid takes the form of 100% loan (i.e. $\theta = 0$), a unit increase in the external aid results in equivalent increase in public debt and the associated adjustment costs. For simplicity, the loan aid is assumed to be provided with no interest rate. To the extent that pubic capital investment involves installation costs, which require domestic resources, a unit increase in the external aid will actually require the government to issue additional public debt to finance the installation component of the investment. This costs get even higher when the external aid takes some amount of loan, since the repayment of the loan aid also accumulates the public debt.

In addition, the government satisfy the following intertemporal budget constraint;

$$\lim_{t\to\infty} D_G\, e^{-r(\frac{D}{K})t} \tag{21}$$

National debt is the sum of private and public debts (i.e. $D = D_P + D_G$). Thus, combining equation (6) and (20), the national budget constraint (the nation's current account) is defined as;

$$\begin{aligned}\dot{D} &= \dot{D_P} + \dot{D_G} \\ &= \left[C + r\left(\frac{D}{K}\right)D_P + \Psi_P(I_P, K_P) - (1-\tau)Y + T\right] \\ &\qquad + \left[\Psi_G(I_G, K_G) + r\left(\frac{D}{K}\right)D_G - \tau Y - T \right. \\ &\qquad \left. - A(\ln(1+\theta) - (1-\theta))\right] \\ &= r\left(\frac{D}{K}\right)D + C + \Psi_P(I_P, K_P) + \Psi_G(I_G, K_G) - Y \\ &\qquad - A(\ln(1+\theta) - (1-\theta))\end{aligned} \tag{22}$$

Equation (22) states that the economy accumulates its national debt to finance interest payments, its consumption, and its total investment on private and public capital, net of output produced and the net external aid received.

3.3. Macroeconomic Equilibrium

The steady-state equilibrium has the characteristic that all real quantities grow at the same constant rate and that the relative price of capital, q, is constant. Thus, the dynamics of the system can be expressed with the stationary variables, normalized by the private capital stock ($c \equiv \frac{C}{K_P},\ k \equiv \frac{K_G}{K_P},\ d \equiv \frac{D}{K_P}$) and q. The equilibrium system can be derived as follows.

Firstly, taking the time derivative of k and substituting (11) and (19) yields;

$$\begin{aligned}\frac{\dot{k}}{k} &\equiv \phi_G - \phi_P \\ &= [\sigma_G + \lambda_{GA}\ln(1+\theta)\,\sigma_A + \lambda_{LA}(1-\theta)\sigma_A]\frac{Y}{K_G} - \delta_G - \left[\frac{q-1}{h_P} - \delta_P\right] \\ &= [\sigma_G + \lambda_{GA}\ln(1+\theta)\,\sigma_A + \lambda_{LA}(1-\theta)\sigma_A]\,\alpha k^{\eta-1} - \frac{q-1}{h_P} - (\delta_G - \delta_P) \qquad (23)\end{aligned}$$

$$\begin{aligned}\rightarrow \dot{k} = [\sigma_G + \lambda_{GA}\ln(1+\theta)\,\sigma_A + \lambda_{LA}(1-\theta)\sigma_A]\,\alpha k^{\eta} - \frac{q-1}{h_P}k \qquad (24) \\ - (\delta_G - \delta_P)k\end{aligned}$$

Secondly, dividing (22) by national debt stock, D, and substituting, (22) can be rewritten as;

$$\begin{aligned}\frac{\dot{D}}{D} &\equiv \phi_D \\ &= r\left(\frac{d}{1+k}\right) \\ &\quad + \frac{1}{d}\Big[\{[\sigma_G + \lambda_{GA}\ln(1+\theta)\,\sigma_A + \lambda_{LA}(1-\theta)\sigma_A] \\ &\quad - [1 + \sigma_A(\ln(1+\theta) - (1-\theta))]\}\alpha k^{\eta} \\ &\quad + \alpha^2\frac{h_G}{2}[\sigma_G + \lambda_{GA}\ln(1+\theta)\,\sigma_A + \lambda_{LA}(1-\theta)\sigma_A]^2 k^{2\eta-1} \\ &\quad + \frac{(q^2-1)}{2h_P} + c\Big]\end{aligned} \qquad (25)$$

Taking the time derivative of d and combining with (11),

$$\frac{\dot{d}}{d} \equiv \phi_D - \phi_P$$
$$= r\left(\frac{d}{1+k}\right) + \frac{1}{d}\Bigg[\{[\sigma_G + \lambda_{GA}\ln(1+\theta)\,\sigma_A + \lambda_{LA}(1-\theta)\sigma_A]$$
$$- [1 + \sigma_A(\ln(1+\theta) - (1-\theta))]\}\alpha k^{\eta}$$
$$+ \alpha^2 \frac{h_G}{2}[\sigma_G + \lambda_{GA}\ln(1+\theta)\,\sigma_A + \lambda_{LA}(1-\theta)\sigma_A]^2 k^{2\eta-1}$$
$$+ \frac{(q^2-1)}{2h_P} + c\Bigg] - \frac{q-1}{h_P} + \delta_P \tag{26}$$

$$\rightarrow \dot{d} = r\left(\frac{d}{1+k}\right)d$$
$$+ \Bigg[\{[\sigma_G + \lambda_{GA}\ln(1+\theta)\,\sigma_A + \lambda_{LA}(1-\theta)\sigma_A]$$
$$- [1 + \sigma_A(\ln(1+\theta) - (1-\theta))]\}\alpha k^{\eta}$$
$$+ \alpha^2 \frac{h_G}{2}[\sigma_G + \lambda_{GA}\ln(1+\theta)\,\sigma_A + \lambda_{LA}(1-\theta)\sigma_A]^2 k^{2\eta-1}$$
$$+ \frac{(q^2-1)}{2h_P} + c\Bigg] - \frac{q-1}{h_P}d + \delta_P d \tag{27}$$

Thirdly, from (9) and (12), the growth rate of consumption is expressed as;

$$\frac{\dot{C}}{C} \equiv \phi_C = \frac{r\left(\frac{d}{1+k}\right) - \beta}{1-\gamma} \tag{28}$$

Taking the time derivative of c and combining with (11),

$$\frac{\dot{c}}{c} \equiv \phi_C - \phi_P$$
$$= \frac{r\left(\frac{d}{1+k}\right) - \beta}{1-\gamma} - \frac{q-1}{h_P} + \delta_P \tag{29}$$

$$\rightarrow \dot{c} = \frac{r\left(\frac{d}{1+k}\right) - \beta}{1-\gamma}c - \frac{q-1}{h_P}c + \delta_P c \tag{30}$$

Finally, rewriting (13) implies;

$$\dot{q} = r\left(\frac{d}{1+k}\right)q - (1-\tau)(1-\eta)\alpha k^{\eta} - \frac{(q-1)^2}{2h_p} + \delta_P q \tag{31}$$

Equation (24), (27), (30) and (31) provide an autonomous set of dynamic

equations in k, d, c and q, from which the evolution of public debt can be derived.

3.4. Steady-State Equilibrium

The economy reaches the steady state when $\dot{k} = \dot{d} = \dot{c} = \dot{q} = 0$, implying that $\frac{\dot{K_P}}{K_P} = \frac{\dot{K_G}}{K_G} = \frac{\dot{D}}{D} = \frac{\dot{C}}{C} \equiv \phi_{SS}$, the balanced growth rate of the economy. The steady state is thus described by;

$$[\sigma_G + \lambda_{GA} \ln(1+\theta)\,\sigma_A + \lambda_{LA}(1-\theta)\sigma_A]\,\alpha(k_{SS})^{\eta-1} - \delta_G = \frac{q_{SS}-1}{h_P} - \delta_P \quad (32)$$

$$\begin{aligned} r_{SS}\left(\frac{d_{SS}}{1+k_{SS}}\right) - \frac{1}{d_{SS}}\Big[&\{1-\sigma_G \\ &+ [(1-\lambda_{GA})\ln(1+\theta) - (1+\lambda_{LA})(1-\theta)]\sigma_A\}\alpha(k_{SS})^{\eta} \\ &- \alpha^2\frac{h_G}{2}[\sigma_G + \lambda_{GA}\ln(1+\theta)\,\sigma_A \\ &+ \lambda_{LA}(1-\theta)\sigma_A]^2(k_{SS})^{2\eta-1} - \frac{(q_{SS}^2-1)}{2h_P} - \bar{c}\Big] \\ &= \frac{q_{SS}-1}{h_P} - \delta_P \end{aligned} \quad (33)$$

$$\frac{r_{SS}\left(\frac{d_{SS}}{1+k_{SS}}\right) - \beta}{1-\gamma} = \frac{q_{SS}-1}{h_P} - \delta_P = \phi_{SS} \quad (34)$$

$$r_{SS}\left(\frac{d_{SS}}{1+k_{SS}}\right)q_{SS} - (1-\tau)(1-\eta)\alpha(k_{SS})^{\eta} - \frac{(q_{SS}-1)^2}{2h_p} + \delta_P q_{SS} = 0 \quad (35)$$

These equations determine the steady-state equilibrium in the following recursive manner. Firstly, equations (32) – (35) jointly determine k_{SS}, q_{SS}, and $r_{SS}\left(\frac{d_{SS}}{1+k_{SS}}\right)$, from which the steady-state growth rate ϕ_{SS} immediately follows. Having determined k_{SS} and $r_{SS}\left(\frac{d_{SS}}{1+k_{SS}}\right)$, the equilibrium stock of debt-capital ratio, d_{SS}, is obtained from (5). Givenk_{SS}, q_{SS}, $r_{SS}\left(\frac{d_{SS}}{1+k_{SS}}\right)$, and d_{SS}, the equilibrium consumption-capital ratio, c_{SS}, is obtained from the current account equilibrium

condition, (30). Provided $r_{SS}\left(\frac{d_{SS}}{1+k_{SS}}\right) > \phi_{SS}$ (which is required for the transversality condition to hold), higher marginal borrowing costs reduce total interest payments raising the consumption-capital ratio. Also, higher installation costs, h_G, reduce the amount of output available for consumption. Because this system is highly non-linear, it need not be consistent with a well-defined steady-state equilibrium with $k_{SS} > 0$, $c_{SS} > 0$. However, the subsequent numerical simulations yield well-defined steady-state values for all plausible specifications of all the structural and policy parameters of the model.

The functional specification of the upward-sloping supply curve to be used is the following. Thus, in absence of any borrowing premium, when $a = 0$, then $r = r^*$, the world interest rate.

$$r_{SS}\left(\frac{d_{SS}}{1+k_{SS}}\right) = r^* + e^{\frac{ad}{(1+k)}} - 1 \tag{36}$$

3.5. Equilibrium Dynamics

Equations (24), (27), (30) and (31) form the dynamics of the system in terms of k, d, c and q. Linearizing these equations around the steady-state values leads to the following equilibrium dynamics;

$$\begin{pmatrix} \dot{k} \\ \dot{d} \\ \dot{c} \\ \dot{q} \end{pmatrix} = \left(\begin{matrix} \frac{\partial eq(24)}{\partial k} & \frac{\partial eq(24)}{\partial d} & \frac{\partial eq(24)}{\partial c} & \frac{\partial eq(24)}{\partial q} \\ \frac{\partial eq(27)}{\partial k} & \frac{\partial eq(27)}{\partial d} & \frac{\partial eq(27)}{\partial c} & \frac{\partial eq(27)}{\partial q} \\ \frac{\partial eq(30)}{\partial k} & \frac{\partial eq(30)}{\partial d} & \frac{\partial eq(30)}{\partial c} & \frac{\partial eq(30)}{\partial q} \\ \frac{\partial eq(31)}{\partial k} & \frac{\partial eq(31)}{\partial d} & \frac{\partial eq(31)}{\partial c} & \frac{\partial eq(31)}{\partial q} \end{matrix} \right)\Bigg|_{k_G=\overline{k_G},\ d=\bar{d},\ c=\bar{c},\ q=\bar{q}} \cdot \begin{pmatrix} k - k_{SS} \\ d - d_{SS} \\ c - c_{SS} \\ q - q_{SS} \end{pmatrix} \tag{37}$$

$$= \begin{pmatrix} a_{11} & 0 & 0 & -\frac{k_{SS}}{h_P} \\ a_{21} & \frac{r'_{SS}(\cdot)d_{SS}}{1+k_{SS}} + r_{SS}(\cdot) - \phi_{SS} & 1 & \frac{(q_{SS}-d_{SS})}{h_P} \\ -\frac{c_{SS}}{(1-\gamma)}\frac{r'_{SS}(\cdot)d_{SS}}{(1+k_{SS})^2} & \frac{c_{SS}}{(1-\gamma)}\frac{r'_{SS}(\cdot)}{(1+k_{SS})} & 0 & -\frac{c_{SS}}{h_P} \\ -\frac{r'_{SS}(\cdot)d_{SS}}{(1+k_{SS})^2} - \eta(1-\tau)(1-\eta)\alpha(k_{SS})^{\eta-1} & \frac{r'_{SS}(\cdot)}{(1+k_{SS})}q_{SS} & 0 & r_{SS}(\cdot) - \phi_{SS} \end{pmatrix}$$
$$\cdot \begin{pmatrix} k - k_{SS} \\ d - d_{SS} \\ c - c_{SS} \\ q - q_{SS} \end{pmatrix}$$

where

$$a_{11} = \eta[\sigma_G + \lambda_{GA}\ln(1+\theta)\,\sigma_A + \lambda_{LA}(1-\theta)\sigma_A]\,\alpha(k_{SS})^{\eta-1} - \delta_G - \phi_{SS}$$

$$a_{21} = -\left[r'_{SS}(\cdot)\frac{d_{SS}^2}{(1+k)^2} + \eta[\sigma_G + \lambda_{GA}\ln(1+\theta)\,\sigma_A + \lambda_{LA}(1-\theta)\sigma_A]\,\alpha(k_{SS})^{\eta-1} + (2\eta-1)\alpha^2\frac{h_G}{2}[\sigma_G + \lambda_{GA}\ln(1+\theta)\,\sigma_A + \lambda_{LA}(1-\theta)\sigma_A]^2(k_{SS})^{2\eta-2}\right]$$

The determinant of the coefficient matrix of (37) can be shown to be positive under the condition that $r_{SS} > \phi_{SS}$ i.e., the steady-state interest rate facing the borrowing country must be greater than the steady-state growth rate of the economy. Imposing the transeversality condition (14), the above-mentioned condition is indeed satisfied. Since (37) is a fourth-order system, a positive determinant implies that there could be zero, two or four positive (unstable) roots. Imposing the following conditions: (i) $-\frac{1}{2} < \gamma < 0$, (ii) $\delta_G \leq \delta_P$, and (iii) $q_{SS} > d_{SS}$, suffices to rule out the case of zero and four positive roots. Note that the conditions (i) – (iii) are sufficient for saddle-point stability. Thus the dynamic system (37) can be shown to be saddle-point stable with two positive (unstable) and two negative (stable) roots, the latter being denoted by ρ_1 and ρ_2, with $\rho_2 < \rho_1 < 0$. The stable solution is of the generic form:

$$k(t) - k_{SS} = B_1 e^{\rho_1 t} + B_2 e^{\rho_2 t} \tag{38}$$

$$d(t) - d_{SS} = B_1 m_{21} e^{\rho_1 t} + B_2 m_{22} e^{\rho_2 t} \tag{39}$$

$$c(t) - c_{SS} = B_1 m_{31} e^{\rho_1 t} + B_2 m_{32} e^{\rho_2 t} \tag{40}$$

$$q(t) - q_{SS} = B_1 m_{41} e^{\rho_1 t} + B_2 m_{42} e^{\rho_2 t} \tag{41}$$

where B_1 and B_2 are arbitrary constants and the vector $(1 \quad m_{2i} \quad m_{3i} \quad m_{4i})'$, $i = 1,2$ (where the prime denotes vector transpose) is the normalized eigenvector associated with the stable eigenvalue, ρ_i. That is, $(1 \quad m_{2i} \quad m_{3i} \quad m_{4i})'$ satisfies:

$$\begin{pmatrix} -\rho_i & 0 & 0 & -\frac{k_{SS}}{h_P} \\ a_{21} & \frac{r'_{SS}(\cdot)d_{SS}}{1+k_{SS}} + r_{SS}(\cdot) - \phi_{SS} - \rho_i & 1 & \frac{(q_{SS}-d_{SS})}{h_P} \\ -\frac{c_{SS}}{(1-\gamma)}\frac{r'_{SS}(\cdot)d_{SS}}{(1+k_{SS})^2} & \frac{c_{SS}}{(1-\gamma)}\frac{r'_{SS}(\cdot)}{(1+k_{SS})} & -\rho_i & -\frac{c_{SS}}{h_P} \\ -\frac{r'_{SS}(\cdot)d_{SS}}{(1+k_{SS})^2} - \eta(1-\tau)(1-\eta)\alpha(k_{SS})^{\eta-1} & \frac{r'_{SS}(\cdot)}{(1+k_{SS})} q_{SS} & 0 & r_{SS}(\cdot) - \phi_{SS} - \rho_i \end{pmatrix} \cdot \begin{pmatrix} 1 \\ m_{2i} \\ m_{3i} \\ m_{4i} \end{pmatrix} = 0 \tag{42}$$

The arbitrary constants B_1 and B_2, appearing in the solutions, (38)-(41), are obtained from initial conditions, specifically that the economy starts out with given initial stocks of capital and debt k_0 and d_0. Setting $t = 0$ in (38) and (40) and letting $\mathrm{d}k_{SS} \equiv k_{SS} - k_0$, and $\mathrm{d}d_{SS} \equiv d_{SS} - d_0$, then B_1 and B_2 are given by:

$$B_1 = \frac{\mathrm{d}d_{SS} - m_{22}\mathrm{d}k_{SS}}{m_{22} - m_{21}}; \quad B_2 = \frac{m_{21}\mathrm{d}k_{SS} - \mathrm{d}d_{SS}}{m_{22} - m_{21}} \tag{43}$$

The constants B_1 and B_2 thus depend on the specific shocks, and once determined, the complete solution for the equilibrium evolution follows from (38)-(41).

4. Numerical Analysis of Transitional Paths

4.1. Permanent Shocks

Further insights into the effects of aid inflow are obtained from analyzing the model numerically. The following numerical simulation is based on calibration of a benchmark economy, using the following parameters representative of a small open economy which starts out from an equilibrium with zero transfers. The parameter values, described in Table 3-2, are conventional and lead to the following plausible

benchmark equilibrium reported in row (0) of Table 3-3.

Table 3-2: Parameter Values for Benchmark Economy

Preference Parameters:	$\gamma = -0.4,\ \beta = 0.04$
Production Parameters:	$\alpha = 0.4,\ \eta = 0.2,\ h_P = 15,\ h_G = 15$
Depreciation Rates:	$\delta_P = 0.05,\ \delta_G = 0.04$
Borrowing Condition:	$r^* = 0.06,\ a = 0.1$
Policy Parameters:	$\tau = 0.15,\ \sigma_G = 0.05$
External Aid Parameters:	$\sigma_A = 0,\ \lambda_{GA} = 0.2,\ \lambda_{LA} = 0.5$
"Traffic Light" Parameters	$\theta = 1$

Table 3-3: Steady-State Values for Permanent Shocks

	k_{SS}	d_{SS}	c_{SS}	q_{SS}	ϕ_{SS}	K_G/Y	K_P/Y	C/Y	D/Y
(0) Benchmark $\sigma_A = 0,\ \theta = 1$	0.2515	0.1057	0.1689	2.0551	2.03%	0.8286	3.2948	0.5565	0.3483
(1) Red Light $\sigma_A = 0.05,\ \theta = 1$	0.2868	0.1357	0.1792	2.0779	2.19%	0.9205	3.2094	0.5751	0.4355
(2) Yellow Light $\sigma_A = 0.05,\ \theta = 0.5$	0.3359	0.1752	0.1653	2.1058	2.37%	1.0445	3.1096	0.5140	0.5448
(3) Green Light $\sigma_A = 0.05,\ \theta = 0$	0.3792	0.2085	0.1482	2.1275	2.52%	1.1509	3.0351	0.4498	0.6328

From (1) and ($c \equiv \frac{C}{K_P},\ k \equiv \frac{K_G}{K_P},\ d \equiv \frac{D}{K_P}$), the key variables are obtained as:

$$\frac{K_G}{Y} = \frac{k}{\alpha(k)^\eta} \tag{44}$$

$$\frac{K_P}{Y} = \frac{1}{\alpha(k)^\eta} \tag{45}$$

$$\frac{C}{Y} = \frac{c}{\alpha(k)^\eta} \tag{46}$$

$$\frac{D}{Y} = \frac{d}{\alpha(k)^\eta} \tag{47}$$

The ratio of public-private capital, k, is 0.2515; the public capital-output ratio, K_G/Y, is 0.8286; the private capital-output ratio, K_P/Y, is 3.2948; the consumption-output ratio, C/Y, is 0.5565; and the debt-output ratio, D/Y, is 0.3483, with the equilibrium growth rate, ϕ_{SS}, being 2.03%. This equilibrium is a reasonable

characterization of a small medium-indebted developing economy experiencing a modest steady rate of growth and having a relatively small stock of public capital.

Row (1) – (3) summarizes the permanent changes to this equilibrium following the aid inflow equal to 5% of the recipient country's GDP. Row (1) describes the permanent effects of "grant" aid inflow. Row (2) describes the permanent effects of aid inflow, which is in the form of grant and loan mixture. Row (3) describes the permanent effects of "loan" aid inflow. The following analysis is based on the conventional assumption that the system starts out from an initial steady-state equilibrium.

In the steady state of Row (3), the ratio of public to private capital increases from 0.8286 to 1.1509, thereby generating a huge investment boom in infrastructure. The increase in the stock of public capital increases the marginal productivity of private capital. Although the aid inflow stimulates consumption through the wealth effect, the higher long-run productive capacity has a greater effect on output, leading to a decline in the long-run consumption – output ratio from 0.5565 to 0.4498. The higher productivity raises the long-run growth rate to 2.52%. The increased accumulation of both public and private capital lead to a higher demand for external borrowing as a means of financing new investment in private capital and the installation costs of public capital. This results in an increase in the steady state debt – output ratio from 0.3483 to 0.6328. However, this higher debt relative to output is sustainable since it is caused by higher investment demand rather than higher consumption demand. The long run increase in the economy's productive capacity (as measured by the higher stocks of public and private capital, and output) ensures that the higher debt is sustainable.

While the above parameters represent a plausible small open developing economy, some of the results are dependent on the characterization. The results of

some sensitivity analysis are summarized in Table 3-4 and Table 3-5.

Table 3-4: Sensitivity to Aid Dependency

	$\sigma_A = 0.01$		$\sigma_A = 0.10$	
	ϕ_{SS}	D/Y	ϕ_{SS}	D/Y
(1) Red Light $\theta = 1$	2.07%	0.3663	2.32%	0.5155
(2) Yellow Light $\theta = 0.5$	2.11%	0.3909	2.64%	0.7119
(3) Green Light $\theta = 0$	2.15%	0.4121	2.87%	0.8650

Table 3-5: Sensitivity to Installation Costs

	$h_P = h_G = 10$		$h_P = h_G = 20$	
	ϕ_{SS}	D/Y	ϕ_{SS}	D/Y
(1) Red Light $\theta = 1$	0.33%	0.8946	3.80%	0.0996
(2) Yellow Light $\theta = 0.5$	0.47%	1.0176	4.02%	0.1974
(3) Green Light $\theta = 0$	0.59%	1.1164	4.19%	0.2762

Table 3-4 considers two alternative specifications of benchmark economy, corresponding to $\sigma_A = 0.01$ (less aid dependent economy) and $\sigma_A = 0.10$ (more aid dependent economy). General trend remains same, but more aid dependent economy can sustain higher, almost twice as much as, debt level, because all of those larger aid inflow is assumed to be productively invested into public capital. Table 3-5 presents how the installation costs associated with private and public capital investment, h_P and h_G, affect the long-run equilibrium. The country with higher installation costs, $h_P = h_G = 20$, can only sustain lower debt – output ratio, at less than 30% of GDP in any cases. Higher installation costs reflects the situations which hinders infrastructure development, such as less development of infrastructure systems, weak capacity of line ministries, internal conflict of the country, fraud, corruption, and so forth, all of

which are common characterization in the less developed countries. Therefore, Table 3-5 indicates that the sustainable debt level is lower for the less developed countries.

The transitional dynamic paths from the benchmark economy (i.e. "no aid") to "Green Light" economy are depicted in Figure 3-3 through Figure 3-8. Figure 3-3 through Figure 3-6 illustrates the transition path for the ratio of private capital – output, public capital – output, consumption – output, and debt – output, respectively. The K_P/Y ratio declines and the D/Y ratio increases monotonically through time. This is because the accumulation of public capital raises the average productivity of private capital, while the accumulation of both capital raises the need to borrow from abroad. Figure 7 illustrates the stable adjustment locus in $k-d$ space, including how k and d increase almost proportionately during the transition.

The transitional paths of the growth rates for ϕ_P, ϕ_G, ϕ_C, ϕ_D, and ϕ_Y toward their common long-run growth rate are shown in Figure 8. The stimulus to public capital raises its initial growth rate to over 5%, after which it declines monotonically. By contrast, private capital adjusts only gradually. The growth rate of output initially reaches more than 2.6%, then declines to the long-run growth rate 2.52%. At first the debt growth rate rises sharply to over 15%, due to the fact that the country suddenly receives the aid in the form of loan, but it monotonically declines through time. This is because the loan aid inflow promotes accumulation of public capital, which raises the average productivity of private capital, while the accumulation of both types of capital raises the need to borrow from abroad. However, as the marginal productivity of private capital declines over time, the accumulation of public and thus private capital slows down, which eventually reduces the growth rate of debt.

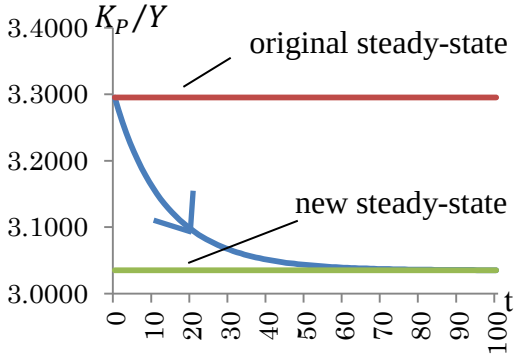

Figure 3-3: Transitional Adjustment from "no aid" to "Green Light" Private Capital-Output Ratio

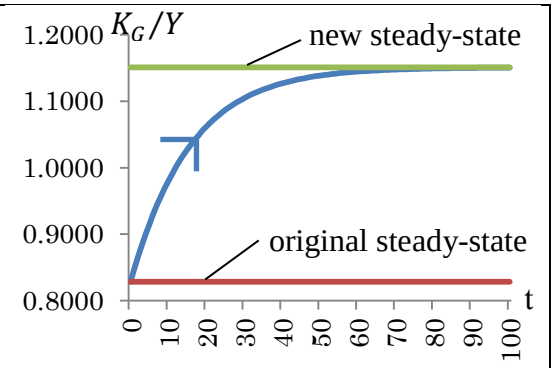

Figure 3-4: Transitional Adjustment from "no aid" to "Green Light" Public Capital-Output Ratio

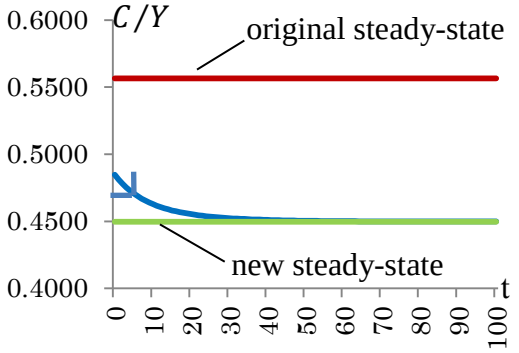

Figure 3-5: Transitional Adjustment from "no aid" to "Green Light" Consumption-Output Ratio

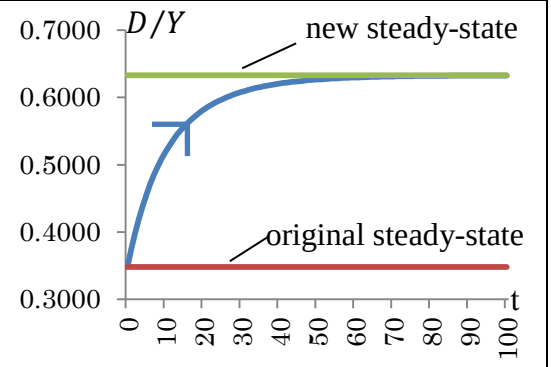

Figure 3-6: Transitional Adjustment from "no aid" to "Green Light" Debt-Output Ratio

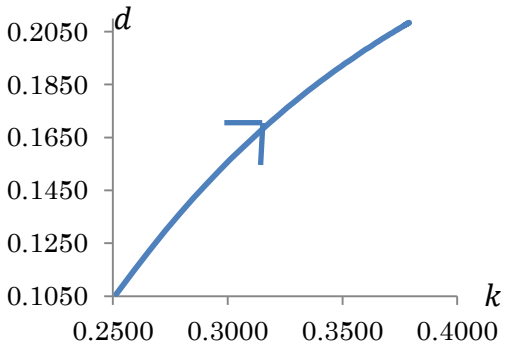

Figure 3-7: Transitional Adjustment from "no aid" to "Green Light" Transitional Adjustment Locus

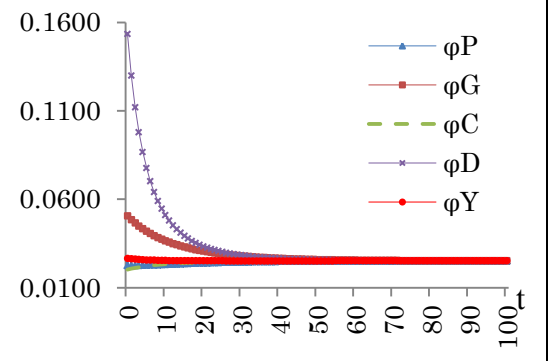

Figure 3-8: Transitional Adjustment from "no aid" to "Green Light" Growth Rates

The transitional dynamic paths from "Green Light" economy to "Red Light" economy are depicted in Figure 3-9 through Figure 3-14.

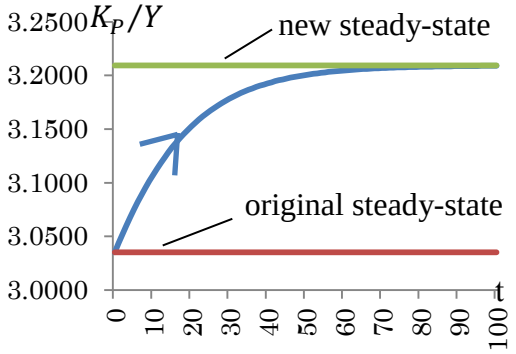

Figure 3-9: Transitional Adjustment from "Green Light" to "Red Light" Private Capital-Output Ratio

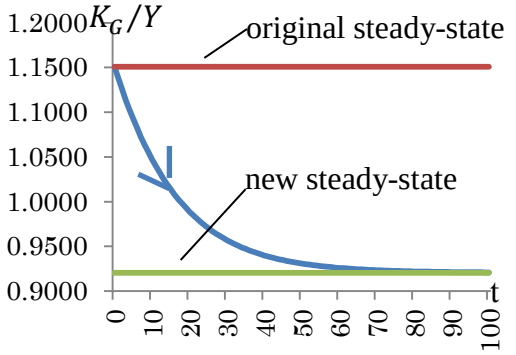

Figure 3-10: Transitional Adjustment from "Green Light" to "Red Light" Public Capital-Output Ratio

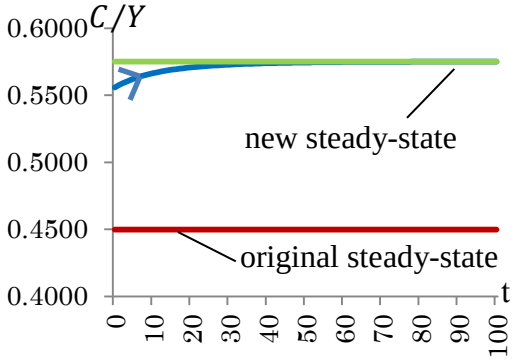

Figure 3-11: Transitional Adjustment from "Green Light" to "Red Light" Consumption-Output Ratio

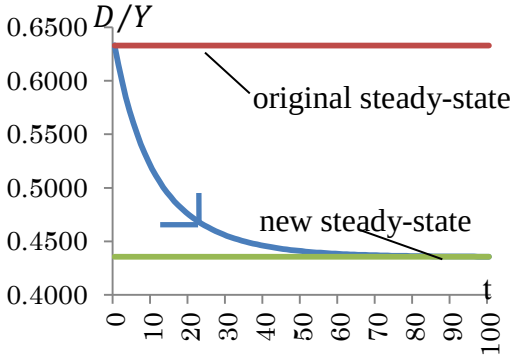

Figure 3-12: Transitional Adjustment from "Green Light" to "Red Light" Debt-Output Ratio

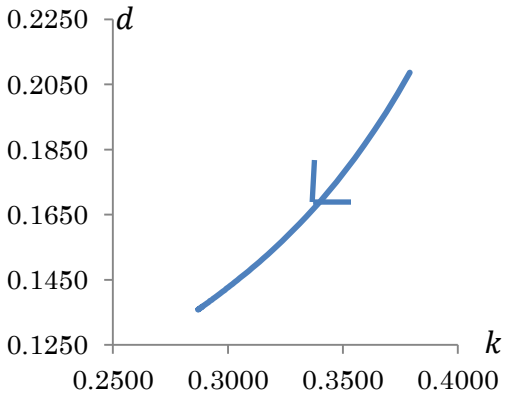

Figure 3-13: Transitional Adjustment from "Green Light" to "Red Light" Transitional Adjustment Locus

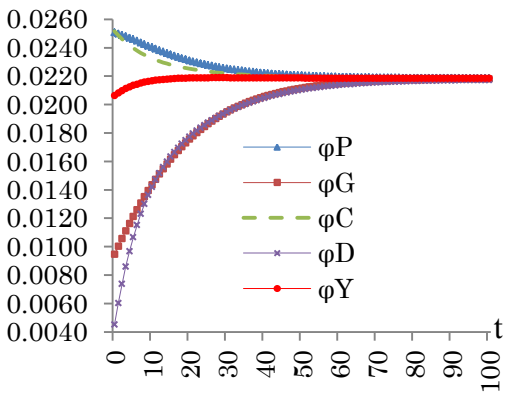

Figure 3-14: Transitional Adjustment from "Green Light" to "Red Light" Growth Rates

In the new steady state under "Red Light", the ratio of public to private capital decreases from 0.3792 to 0.2868, which leads to lower marginal productivity of

private capital. The decreased accumulation of both public and private capital lead to a lower demand for external borrowing as a means of financing new investment in private capital and the installation costs of public capital. This results in decrease in the steady state debt – output ratio from 0.6328 to 0.4355. However, the lower productivity decreases the long-run growth rate to 2.19%. Figure 3-9 through Figure 3-12 illustrates the transition path for the ratio of private capital – output, public capital – output, consumption – output, and debt – output, respectively. As opposed to Figure 3-3 through Figure 3-6, the K_P/Y ratio increases and the D/Y ratio declines monotonically through time. This is because the decrease of public capital lowers the average productivity of private capital, while decrease of both capital lowers the need to borrow from abroad. Figure 3-13 illustrates the stable adjustment locus in $k - d$ space, including how k and d decrease over time.

The transitional growth rates are depicted in Figure 3-14. The growth rate of debt plunges to less than 0.5%, because the aid takes the form of grant instead of loan. Over time, as the marginal productivity of private capital improves, the need for external borrowing raises the growth rate of debt to the long-run sustainable rate, although the debt – output ratio declines monotonically.

4.2. Temporary Shocks

While analyses in 4.1. explore the permanent effect of grant/loan aid inflow, the realty which the recipient countries are faced up with is volatile aid modality changes. In other words, "traffic light" turns to "red" for several periods then returns to "green", for instance. Under such situations, a recipient country's transitional path toward steady state becomes more complicated than that of the permanent shocks.

This section assumes that "traffic light" for recipient country is initially "green", but turns into "red" for the five periods, then returns to "green". The

transitional dynamic paths are depicted in Figure 3-15 through Figure 3-20.

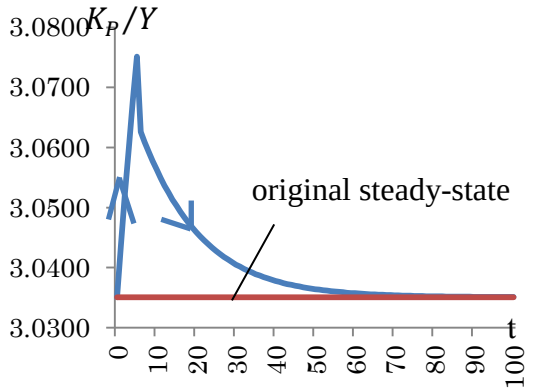

Figure 3-15: Transitional Adjustment from "Green Light" to "Red Light" then to "Green Light" Private Capital-Output Ratio

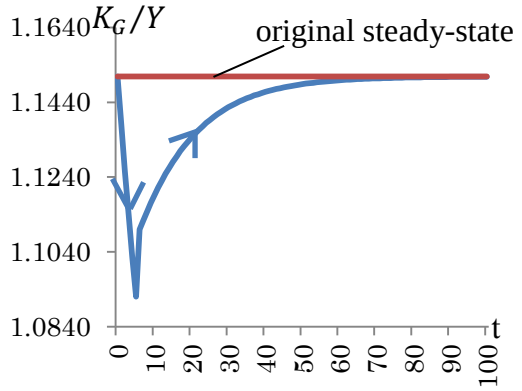

Figure 3-16: Transitional Adjustment from "Green Light" to "Red Light" then to "Green Light"Public Capital-Output Ratio

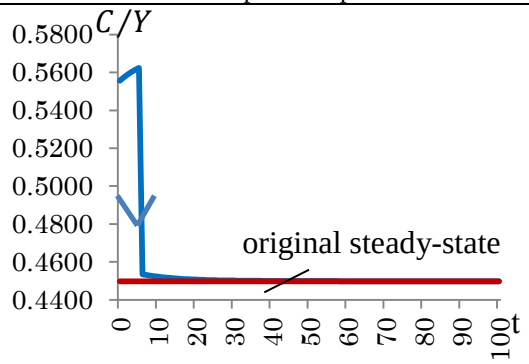

Figure 3-17: Transitional Adjustment from "Green Light" to "Red Light" then to "Green Light" Consumption-Output Ratio

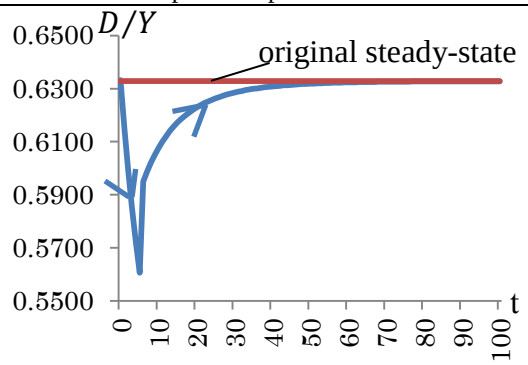

Figure 3-18: Transitional Adjustment from "Green Light" to "Red Light" then to "Green Light" Debt-Output Ratio

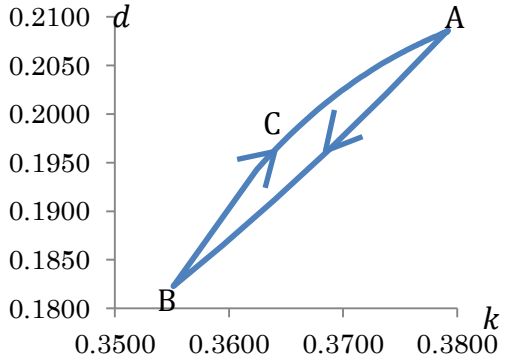

Figure 3-19: Transitional Adjustment from "Green Light" to "Red Light" then to "Green Light" Transitional Adjustment Locus

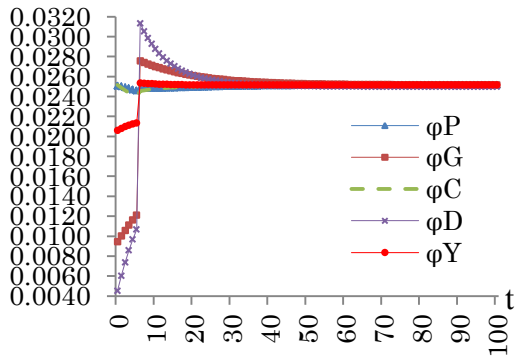

Figure 3-20: Transitional Adjustment from "Green Light" to "Red Light" then to "Green Light" Growth Rates

In contrast to the permanent shocks depicted in previous section, the adjustments are characterized by reversal transitional dynamics. Suppose that the economy starts out from the equilibrium point A in Figure 3-19. During the period of "Red Light", the grant aid inflow reduces the debt and public capital investment, which is represented by the moving along the locus AB. Once the "traffic light" changes back to "Green Light", the inflow of loan aid, which is more inclined to be invested into public capital than the grant aid, raises the public capital stock by increasing the debt. Thus, the economy starts increasing the debt and public capital along the locus BCA.

Figure 3-15 through 3-18 illustrate the dynamic time paths for the ratio of private capital – output, public capita – output, consumption – output, debt – output ratios, respectively. These all show the reversal transition from "Red Light" period to "Green Light" period.

Figure 3-20 illustrates the growth rates. After the five periods of receiving grant aid during "Red Light", the growth rates increases abruptly back to the higher equilibrium level. Its volatility seems harmful to the country's economy.

5. Policy Implications and Way Forward

This paper addresses the macroeconomic impact of the traffic light system. There are three main policy implications derived.

Firstly, the numerical analyses indicate the possibility that the current thresholds for external borrowing under the traffic light system could be sub-optimal for some countries with high potential to achieve economic growth by utilizing debt-financed investment. This is represented by the steady-state value of debt – output ratio, namely 0.4355 for "Red Light", 0.5448 for "Yellow Light", and 0.6328 for

"Green Light", all of which is larger than the thresholds under the traffic light system. For instance, under the traffic light system, debt – GDP threshold for the country with strong policy is set at 50%, and therefore the country's "traffic light" may be classified as "Red" if the country's debt level exceeds 50%. On the other hand, the numerical analyses show that the steady-state value of debt – output ratio for "Green Light" is 63.28%, and thus it indicates that the country can sustainably utilize external borrowing beyond the current "50%" threshold. Therefore, the traffic light system might be unduly constraining the ability of developing countries to finance their development goals, and thus the system needs to be reviewed in terms of validating feasibility of the thresholds. Desirably, the debt thresholds should be calibrated, based on country specific characteristics. For instance, threshold for the country with high potentiality to grow further and the country moving toward its growth ceiling should be different, even if both countries' debt burden indicators are at same level. Use of country-specific information is explored in IMF and the World Bank (2012), which states that IMF and the World Bank would utilize the use of judgment based on country specific information when assessing the risk of debt distress instead of setting country specific thresholds at this moment, but that country specific thresholds based on default probability could be incorporated in future if it approves useful.

Secondly, transient traffic light forces the developing countries into a difficult macroeconomic management, since they need to adjust frequently capital investment policy before the productive capacity of public capital is achieved, since capital investment takes time to be materialized, as indicated in the transitional paths. Therefore, the traffic light system should be refined in terms of securing stability. Volatility is one of the inevitable aspects of the traffic light system, since it is designed, based on both thresholds of CPIA scores and the debt burden indicators, which result in threshold effect. Namely, small changes around the thresholds can lead to discrete

jumps in the traffic light, such as “green” to “red” in some extreme cases. Alternatively, the countries with significantly different level of CPIA or the debt could be classified into the same traffic light category. In this case, the risk of debt distress may be over-estimated for the countries with “better” CPIA score or the debt level, and may be under-estimated for the countries with “worse” CPIA score or the debt level. A s long as the traffic light system is “operational framework”, such kind of volatility cannot be removed systematically. Therefore, it will all come down to how the judgment by IMF and the World Bank staff can alleviate threshold effect. What IMF and the World Bank can do is to develop and provide ex-ante guidance which illustrate how the country specific information is taken into consideration in determining the traffic light.

Thirdly, higher installation costs of capital investment reduces the sustainable debt – output ratio. Those bottlenecks of the developing countries typically stem not from the specific sectors but from the overall system, such as economic management, cost of doing business, and governance structures. Therefore, the external aid needs to take the comprehensive approach, which eventually enhances the country’s debt absorptive capacity.

There are three possible directions to move the research further in future.

Firstly, since the model constructed in this paper treats the traffic light and the volume of aid inflow as given, the natural extension is to endogenize those variables. Namely, future model should be constructed to analyze how the economy reaches to the long-run equilibrium while receiving grant aid when the debt level is within the defined threshold and receiving loan aid when the debt exceeds the limit. Another assumption to be relaxed is that the aid volume is not necessarily proportional to the GDP of the developing country. Practically, after reaching some economic level, the developing country receives less aid toward its graduation from aid recipients status.

Therefore, the assumption behind the current model is justifiable when considering the low income countries, but it may not be applicable to the middle income countries. One of the potential difficulties of this direction is that it is uncertain whether there exist a steady-state solution, because the traffic light system and income categories casts a discontinuous constraint on external borrowing in the model.

Secondly, the model constructed in this paper focuses on the effects of the traffic light on the economic performance of small open developing economy, based on the assumption that there is no feedback effect from such economy to donor economies. However, the traffic light simultaneously affects number of developing economies, and thus the collective feedback effects on the donor economies may no longer be negligible, especially in a sense that the growing demand for grant aid, instead of loan aid, may worsen the financial difficulties of international aid budget in donor economies. Hence, a natural extension of this analysis is to consider the aid in a multi-country growth equilibrium setting.

Thirdly, the model explored in this paper does not take stochasticity into consideration. The current model assumes that the public capital enhances productivity of the private capital in deterministic manner, but there should be an unmanageable productivity shock, such as poor performance of public infrastructures, unreasonable weather, natural disaster, financial crisis of other countries, and so forth. For instance, floods hit the Thailand's economy in 2011, and the blackout of electricity occurred in July 2012 in India affected seriously the private sector's economic activities. Thus, stochastic productivity shock is indispensable factor in analyzing the developing countries economy.

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